

**Oleg Krol**

**METAL CUTTING IN TASKS FOR  
MACHINE TOOL DESIGNERS**

**Monograph**

**Prof. Marin Drinov Academic Publishing House of Bulgarian  
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#### **METAL CUTTING IN TASKS FOR MACHINE TOOL**

**DESIGNERS:** monograph / Krol O. – Sofia: Prof. Marin Drinov Academic Publishing House of Bulgarian Academy of Sciences, 2022. – 138 p.: Table 6. Figure 28. Bibliogr. 105 names. English language.

In the monograph, the theoretical and logical bases of solving problems related to the research of the cutting process of structural steels and alloys are considered. The approach to the study of cutting tool geometry based on a logical analysis chain a research of cutting wedge geometrical parameters from the cutting scheme construction through the coordinate planes to the angles of a cutting tool is carried out. Problem solution on determining kinematic angles of cutters when machining curvilinear contour on NC machine is considered. Analysis of various physical phenomena occurring at metal excess layer removal from the workpiece is performed. Study of physical phenomena is based on the concept of the term "Cutting" as a set of simultaneously performed processes of elastic-plastic deformation of the cut layer, cutting tool wear and blunting as well as the formation of the machined surface of the part. The peculiarity of the cutting tool wear process taking into account the simultaneous effect of abrasive, adhesive, thermal and oxidative effects in the process of cutting is noted. The analytical apparatus and algorithms of optimum cutting modes search with the use of linear and geometrical programming methods and Lagrange multipliers method are given. A software-methodological complex geometrical programming for solving nonlinear problem with components in positive posinomials form is implemented. A software-methodological complex on based Lagrange multipliers method for determine a qualitative picture of cutting processes when the structure and modes change is implemented.

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Prof. Marin Drinov Academic Publishing House of Bulgarian Academy of Sciences  
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## INTRODUCTION

The professional training of mechanical engineers is inextricably linked with the study of the principles of operation of complex mechanisms and machine components. The design of automated machine modules and the whole range of physical, chemical, mechanical and other phenomena of the cutting process are forming the basis of the course "Theory of cutting, physical and thermal processes in technological systems".

Machine tool designers has to face a number of problems where the knowledge of cutting theory is used. Thus, the productivity and production cost of the technological process are determined by the time it takes to perform individual operations and depends on the cutting modes set on them. The conscious appointment of the cutting mode is impossible without knowledge of basic laws for the productive cut, based on the processes that occur in the deformation zone and on the contact surfaces of the tool [1-3].

With precision calculations based on the rigidity of the technological system (machine-fixture-tool-part), one must be able to determine the cutting forces and know what they depend on the direction of their action [4].

In addition to the above, you can specify a number of interactions between the sections and the theory of cutting and other disciplines: reliability of the machine system functioning – wear and tool resistance, etc.; part shape errors – thermal phenomena, etc.

As for the design of a cutting tool, already at its initial stages (the choice of optimal geometric parameters – rake and clearance angles, tool cutting edge angle, etc.) it is necessary to know the essence of the physical processes occurring on the contact surfaces of the tool.

Such interactions for the instrumental section are:

- choice of the tool material type – the features of contacting the pair "tool – material being processed";

- purpose of the criterion for dulling and allowable tool wear is the physical nature and quantitative patterns of wear.

In the research and study of the physical mechanisms underlying the process of cutting metals, one has to deal with a number of typical logical patterns. It is advisable to develop the logic of solving machining problems through the procedures for solving problems.

This monograph consists of eight sections, each of which contains brief theoretical information, a detailed solution of a typical problem and a set of tasks for independent solution.

# 1. CUTTING TOOL, ITS STATIC AND GEOMETRIC PARAMETERS OF THE WORKING PART

## 1.1. Brief theoretical information

Any cutting tool – a cutter, milling cutter, drill bit and other bladed tool designed to penetrate the workpiece and separate the excess material layer, has a cutting wedge as the main functional element. GOST 25721-83 "Cutting" and GOST 25751-83 "Cutting tools" establish terms and definitions of general concepts related to all types of cutting.

The main functional element is the tool blade – a wedge-shaped element of the cutting tool for penetrating the workpiece material and separating the excess layer (Fig. 1) [5, 6]. The face surface of the blade ( $A_\gamma$ ) – surface in contact with the cut layer and chips. Major flank surface of the blade ( $A_\alpha$ ) – the surface in contact with the workpiece surface.

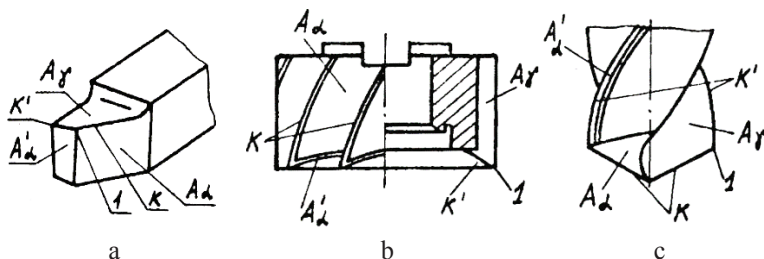


Fig.1. Geometric elements of the tool blade:

a – lathe tool; b – end mill; c – spiral drill;  $A_\gamma$  – the face surface of the blade;  $A_\alpha$  – major clearance surface;  $K$  – major cutting edge;  $A'_\alpha$  – minor clearance surface of the blade;  $K'$  – minor cutting edge; 1 – corner

Cutting edge ( $K$ ) – the edge of the tool blade, formed by the intersection of the face and clearance surfaces.

The corner of the blade (1) is the section of the cutting edge at the point of cutting sections of the two clearance surfaces.

To determine the numerical values of the angular parameters of the blade elements, coordinate systems and coordinate planes are used.

Tool-in-hand coordinate system (TCS) – a rectangular coordinate system with the origin at the corner of the blade, oriented relative to the geometric elements of the cutting tool taken as the base (Fig. 2, a).

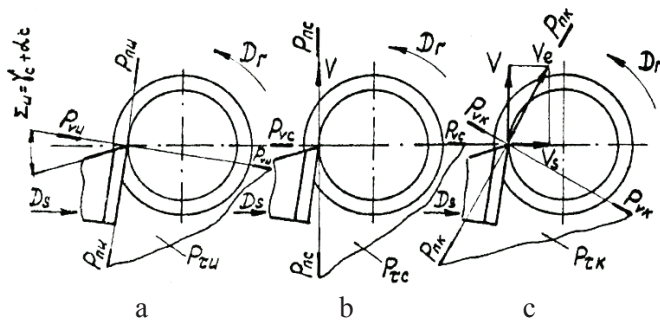


Fig.2. Coordinate systems and coordinate planes:

a – tool cutting; b – static; c – working;

$D_r$  – the main cutting movement;  $D_f$  – feed motion;  $V$  – speed of the main cutting motion;  $V_f$  – feed motion speed;  $V_e$  – speed of the resulting cutting motion;  $P_v$  – reference plane;  $P_n$  – major cutting edge plane;  $P_\tau$  – major orthogonal plane

Static coordinate system (Setting system, SCS) – a rectangular coordinate system with the origin at the considered point of the cutting edge, oriented relative to the direction of the speed of the primary motion of the cutting (Fig. 2, b).

Tool-in-use (kinematic) coordinate system (KCS) – a rectangular coordinate system with the origin at the considered point of the cutting edge, oriented relative to the direction of the speed of the resultant cutting motion (Fig. 2, c).

In the manufacture and control of the tool, it is advisable to use the Tool Coordinate System (TCS), and after mounting the cutting tool on the machine – SCS. When analyzing the parameters of the blade elements in the cutting process, especially when working with high feeds – the KCS system. Most often, in the technical literature, the angles of the cutting tool blade are provided in the SCS, but to simplify the recording, they are designated without additional indices indicating the adopted coordinate system ( $\gamma$ ,  $\alpha$ ,  $\varphi...$ ).

On Fig. 3 shows the coordinate planes, angles and elements of the blade of a lathe tool in a setting system.

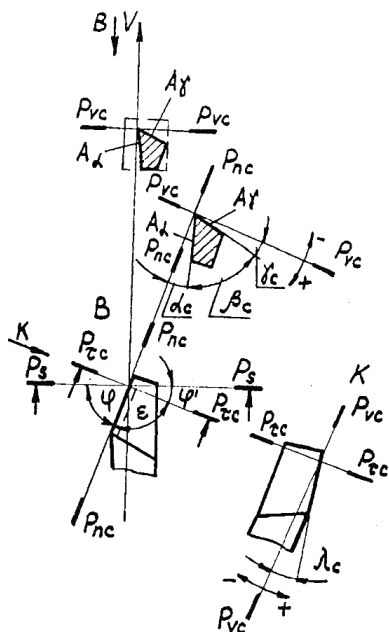


Fig. 3. Cutter angles in static

The tasks of this section are formulated according to the principles of test evaluation and should include not only the choice of one of the options, but also be presented in the form of a cutting scheme [7-9]. The scheme

(fragment) for one of the types of cutting (if it is not specified, then for an arbitrary cutting option) taking into account the required definitions in accordance with the standard – GOST 25721-83 "Cutting", brief explanations and justifications for the answer.

The tasks of this section according to the type of machine less knowledge control cards are formulated. However, the answers to the questions posed should include not only the choice of one of the task positions, but also be presented as a cutting pattern (fragment) for one of the types (if it is not specified, then for arbitrary) cutting; required definitions in accordance with the standard GOST 25721-83 "Cutting".

### ***1.2. Typical task 1. Cutter geometry in static***

*Statement of task 1.* Which of the named angles of the cutter can have a value less than  $0^\circ$ :

$$1 - \gamma_c; 2 - \varphi_c; 3 - \lambda_c; 4 - \varepsilon_c$$

*The solution of the problem.* Let us drawing the cutting scheme (Fig.4). Since the type of processing is not given, let us consider the operation of cutting off the workpiece. We give the definition of the listed angles:

1) static rake angle  $\gamma_c$  – angle in the static orthogonal plane  $P_{\tau c}$  between the face surface of the blade and the static reference plane  $P_{vc}$ ;

2) static edge angle (in plane)  $\varphi_c$  – angle in the static reference plane  $P_{vc}$  between the static cutting edge plane  $P_{nc}$  and the working plane  $P_f$ . The working plane  $P_f$  is the plane in which the directions of the speeds of the primary motion and the feed motion are located;

3) static edge angle of inclination  $\lambda_c$  – the angle in the static cutting edge plane  $P_{nc}$  between the cutting edge  $K$  and the static reference plane  $P_{vc}$ ;

4) static angle at the corner in the plane  $\varepsilon_c$  – angle in the static reference plane  $P_{vc}$  between the trace of the static cutting edge plane  $P_{nc}$  and the minor cutting edge  $K'$ .

Angle  $\gamma_c$  can be taken as positive (Fig. 4), and a negative value. The latter characterizes such a position of the face surface  $A_f$  in which it is located above the static reference plane  $P_{vc}$  and the corner of the cutter is the lowest point of the face surface.

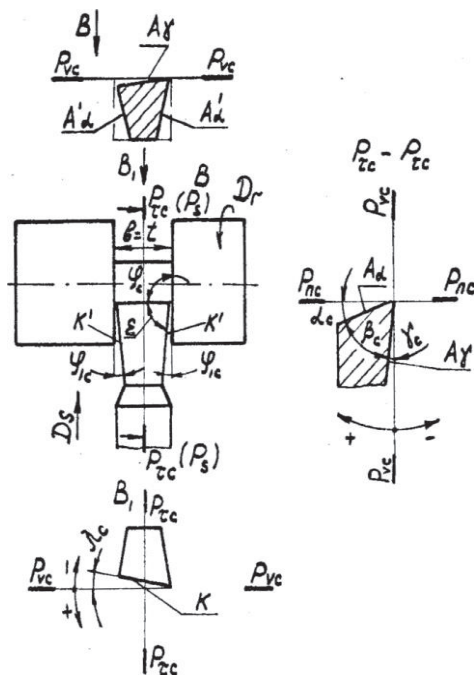


Fig. 4. Scheme of cutting off the workpiece

The angle  $\varphi_c$  cannot take a negative value, since then the tool major cutting edge will coincide with the feed direction and the process of cutting off the workpiece will be unrealizable.

The angle  $\lambda_c$ , unlike  $\gamma_c$  has a different way of counting the sign and may be less than  $0^\circ$ .

With a negative angle  $\lambda_c$ , the volume of the cutting wedge is decreases. (Fig. 4). The angle  $\varepsilon_c$  cannot be less than  $0^\circ$ , since then the major and minor cutting edges will coincide and the concept of a cutting wedge does not make sense.

Answer: the angles  $\gamma_c$  and  $\lambda_c$  can be less than  $0^\circ$ .

### ***1.3. Tasks for independent solution***

1.2.5. The rake angle is considered positive when:

1 – cutting angle  $\delta_c$  (4+5, Fig.5) less than  $90^\circ$ ?

2 – cutting angle  $\delta_c$  (4+5, Fig.5) greater than  $90^\circ$ ?

1.2.6. Which of the indicated angles (Fig. 4) can be equal to zero:

1 –  $\gamma_c$

2 –  $\alpha_c$ ?

3 –  $\lambda_c$ ?

4 –  $\beta_c$ ?

5 –  $\gamma_{lc}$ ?

1.2.7. According to Fig. 5 and the given numerical designations of the cutter angles, indicate the symbolic designations of the corresponding angles:

1 –

5 –

2 –

6 –

3 –

7 –

4 –

8 –



1.2.11. Which of the named angles of the straight-turning tool can be:

I – equal to 0?

II – less than 0?

1 –  $\gamma_c$

1 –  $\gamma_c$

2 –  $\alpha_c$

2 –  $\alpha_c$

3 –  $\varphi_c$

3 –  $\varphi_c$

4 –  $\lambda_c$

4 –  $\lambda_c$

1.2.12. How will the face  $\gamma_c$  and clearance  $\alpha_c$  angles change at the peripheral points of the cutting edge if the corner of the cutter is set on the center line and the angle  $\lambda_c$  is positive:

1 –  $\gamma_c$  will increase?

4 –  $\alpha_c$  will decrease?

2 –  $\gamma_c$  will decrease

5 –  $\gamma_c$  will not change?

3 –  $\alpha_c$  will increase

6 –  $\alpha_c$  will not change?

1.2.13. The angle of cutting edge inclination serves:

1 – to reduce the friction of the cutter clearance surface on the detail surface?

2 – for chip flow direction?

3 – to reduce friction in the contact zone of the minor surface and machined surface?

1.2.14. Positive cutting edge inclination angle is recommended:

1 – when finishing?

2 – when roughing?

1.2.15. When assigning the value of the cutter face angle, take into account:

1 – mechanical qualities of the processed material?

2 – the quality of the material of the cutting tool?

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## 2. CUTTING TOOL AND ITS KINEMATIC GEOMETRIC PARAMETERS OF THE WORKING PART

### *2.1. Brief theoretical information*

In contrast to the static processing scheme, working angles are considered in the cutting process [5, 10, 11]. The logic of the transition to kinematic angles as follows. In the cutting process, a distinction is made between the major cutting motion (rotational motion of the workpiece during turning), the feed motion made by the tool, and the resultant motion made by each point of the cutter cutting edge [12-14].

Each motion is described by its trajectory: circle – for the major cutting motion; straight line – for feed motion; helical line (for transverse turning – it is an Archimedean spiral) – for the resultant motion and is characterized by its speed:  $V$  – velocity vector of the major cutting movement;  $V_f$  – feed rate vector;  $V_e$  – the velocity vector of the resultant motion.

The set of the resultant motions trajectories performed by each point of the cutting edge forms the cutting surface (helical surface for longitudinal turning). Then the kinematic cutting edge plane is tangent to the cutting surface. Changing the position of the cutting edge plane, in comparison with the static one, leads to a change in the reference plane (which is always perpendicular to the cutting plane), which, in turn, leads to a change in the major angles of the cutter  $\gamma_c$  and  $\alpha_c$ .

Thus, the main reasons for considering kinematic angles are the presence in the cutting process of a complex shaping motion and the rotation of the cutting plane [15, 16]. Changing the angles of the cutter

When solving problems, you should first display the cutting pattern; to determine by what value the working (kinematic) angles will change in comparison with the static ones; evaluate the nature of the influence of the cutter geometry on physical phenomena during cutting.

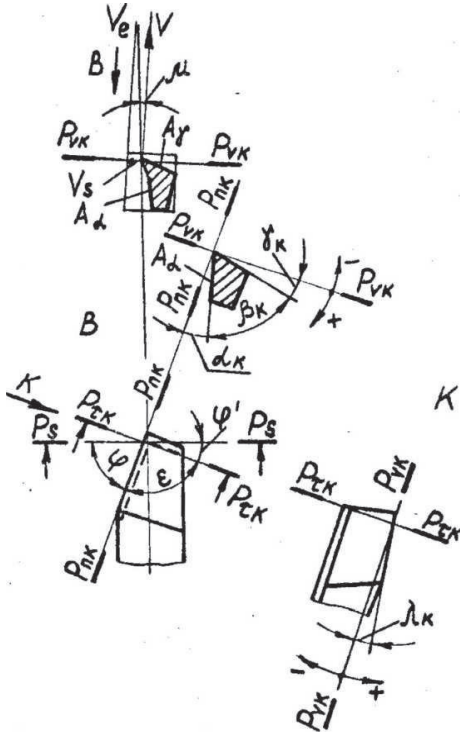


Fig. 6. Kinematic cutter angles

## 2.2. Typical task 2. Cutter geometry in kinematic

Determine the actual (kinematic) major angles  $\alpha_k$  and  $\gamma_k$  at the threaded cutter when cutting the M10 thread, if  $\alpha_c = 15^\circ$ ;  $\gamma_c = 2^\circ$ . Where are these angles greater – at the outer or at the inside diameter of the thread?

*The solution of the problem.* Let us drawing the threading pattern (Fig. 7). As a result of taking into account the motion of the feed  $f$  and the resulting trajectory of motion, characterized by the velocity vector  $V_e$ , the cutting plane  $P_n$  in statics rotates by the angle of helix rise  $\tau_{tr}$  in transverse (*tr*) section from measured in the working plane. Then, by definition, the kinematic clearance angle  $\alpha_k$  will decrease in comparison with the static one  $\alpha_c$  by the value  $\tau_{tr}$ . To determine value  $\tau$  need to deploy in a plane one turns of the circle and one turn of the helix, obtained in one revolution of the workpiece [17-19]. From the resulting right-angled triangle defined through the values of feed and diameter of the workpiece  $\text{tg } \tau_{tr} = f / \pi d$ .

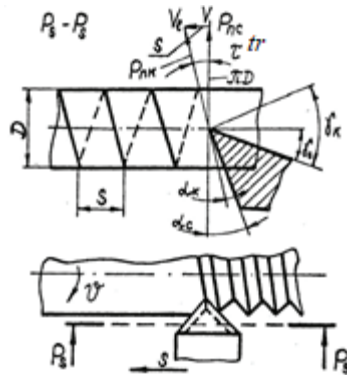


Fig. 7. Thread cutting pattern

Hence the value of the kinematic clearance angle  $\alpha_k$  in the transverse secant plane (for this case, the transverse secant plane coincides with the

work plane  $\alpha_k^{tr} = \alpha_c^{tr} - \tau_{tr}$ . To determine principal clearance angles, which are measured in the major secant plane, you need to find an appropriate relationship between them. For this, we look at three sections of the cutter (Fig. 8). Ratio between major clearance  $\alpha^N$  and clearance angle in transverse cross section  $\alpha^{tr}$  determined by the following scheme:

$$\operatorname{tg} \alpha^{tr} = ab / ac; \operatorname{tg} \alpha^N = ae / ac;$$

$$\operatorname{tg} \alpha^N / \operatorname{tg} \alpha^{tr} = ae / ab = \sin \varphi;$$

$$\operatorname{tg} \alpha^N = \operatorname{tg} \alpha^{tr} \cdot \sin \varphi.$$

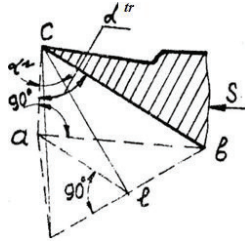


Fig. 8. Cutter cross section

It can be assumed that a similar relationship will be observed between the helix lead angles  $\tau$  in the transverse and major sections, i.e.  $\operatorname{tg} \alpha^N = \operatorname{tg} \alpha^{tr} \cdot \sin \varphi$ . Then kinematic major clearance angle  $\alpha_k^N$  is equal to:

$$\alpha_k^N = \alpha_c^N - \operatorname{arctg} \frac{f}{\pi d} \cdot \sin \varphi.$$

In turn, the major face angle is equal to:

$$\gamma_k^N = \gamma_c^N + \operatorname{arctg} \frac{f}{\pi d} \cdot \sin \varphi.$$

For a threaded cutter dedicated to cutting metric threads, the dimensions established by GOST 8724-61 and GOST 24705-81 are characteristic. The angle of the thread profile, and hence the major angle

in the plan  $\varphi = 60^\circ$ . The thread pitch  $P_{thr}$  is equal to the tool feed, and according to the designation M10 – a thread with a large pitch and a nominal value of the outer diameter  $d_a^{thr}$  equal to 10 mm. For this diameter according to the Table, GOST 8724-81 determine the pitch  $P_{thr} = 1.5$  mm, and, accordingly, the feed  $f = 1.5$  mm/rev.

We determine the values  $\alpha_{ka}^N$  and  $\gamma_{ka}^N$  at the outer diameter of the thread from the relations

$$\alpha_{ka}^N = 15 - \arctg \frac{1.5}{3.14 \cdot 10} \cdot \sin 60^\circ = 12.3^\circ;$$

$$\gamma_{ka}^N = 2 + 2.7 = 4.7^\circ.$$

In order to answer the question of where this angle is greater, it is necessary to find the ratio between the outer and inner diameters of the thread. For pitch  $P_{thr} = 1.5$  mm inner diameter  $d_i = d - 2 + 0.376 = 8.376$  mm. Then at inner diameter:

$$\alpha_{ki}^N = 15 - \arctg \frac{1.5}{3.14 \cdot 8.376} = 12.2^\circ;$$

$$\gamma_{ki}^N = 2 + 2.8 = 4.8^\circ.$$

For the outer diameter of the thread, the major clearance angle is greater ( $\alpha_{ka}^N = 12.3^\circ$ ), and the major rake angle is less ( $\gamma_{ka}^N = 4.7^\circ$ ), than the inner diameter of the thread ( $\alpha_{ki}^N = 12.2^\circ$ ;  $\gamma_{ki}^N = 4.8^\circ$ ).

### ***2.3. Typical task 3. Cutter angles when machining a curved surface***

A curved contour of a part on a CNC lathe is processed. The process by a cutter equipped with a triangular-shaped non-throwaway insert at a constant feed is carried out. Plan angles are  $\varphi = 100^\circ$ ;  $\varphi_1 = 20^\circ$ . Show the

direction of motion of the feed at the points of processing the curved contour and calculate the kinematic angles in the plan [20-22].

The solution of the problem. Let's drawing the scheme of the concave lifting of the contour with the right feed (the X-coordinate increases in the II quadrant). Let's fix the direction of the feed rate at points: B, A, C of the desired contour (Fig. 9).

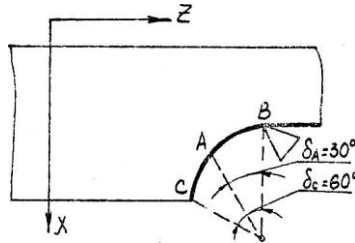


Fig. 9. Contour sketch of processing

Unlike manual machines, a curved contour processing on CNC machines is characterized by a constantly changing position of the working plane  $P_f$  in which the feed vector  $D_f$  is located.

At point B the kinematic and static (tooling,  $t$ ) angles in the plan will be equal to  $\varphi_{kB} = \varphi_{tB} = 100^\circ$ ;  $\varphi_{1kB} = \varphi_{1tB} = 20^\circ$ .

At points A and C, by changing the position  $D_f$  and  $P_f$  the angles will change by the lead angle  $\delta$ :

$$\varphi_{kA} = \varphi_{tA} - \delta_A = 100^\circ - 30^\circ = 70^\circ;$$

$$\varphi_{1kA} = \varphi_{1tA} - \delta_A = 20^\circ + 30^\circ = 50^\circ;$$

$$\varphi_{kC} = \varphi_{tC} - \delta_{Ac} = 100^\circ - 60^\circ = 40^\circ;$$

$$\varphi_{1kC} = \varphi_{1tC} + \delta_C = 20^\circ + 60^\circ = 80^\circ.$$

While their sum remains constant:  $\varphi_k + \varphi_{1k} = 120^\circ$ .

Fig. 10 shows the motion vectors of feeds at points B, A and C.

The kinematic angles at these contour points take on the values:

$$\varphi_{kB} = 100^0; \varphi_{1kB} = 20^0; \varphi_{kA} = 70^0; \varphi_{1kA} = 50^0; \varphi_{kC} = 40^0; \varphi_{1kC} = 80^0.$$

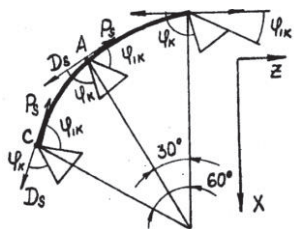


Fig. 10. Camming contour processing

## 2.4. Tasks for independent solution

2.4.1. When setting the tool corner below the centerline in the case of a boring operation:

- 1 – the rake angle decrease?
- 2 – the rake angle increase?
- 3 – the clearance angle will decrease?
- 4 – the wedge angle increase?
- 5 – the clearance angle increase?
- 6 – the wedge angle decrease?
- 7 – the wedge angle will not change?

2.4.2. Determine the kinematic major angles  $\alpha_k$  and  $\gamma_k$  of the threaded cutter for threading, the designation of which is given in Table 1. Where are these angles greater: at the outer diameter or at the inner diameter of the thread?

Table 1

**Determination of kinematic angles. Assignment options**

Variant number	1	2	3	4	5	6	7	8	9
Designation thread	M6	M8	M12	M16	M20	M24	M30	M36	M42
$\alpha_c^0$	8	9	10	11	12	13	14	16	17
$\gamma_c^0$	0	1	3	4	5	6	7	8	9

2.4.3. On a CNC lathe a camming part contour (camming contour sketch and cutting process data are presented in Table.2). Show at the processing points  $A$ ,  $B$  and  $C$  of the contour, motion feed direction and calculate the kinematic angles in plan  $\varphi_k$  and  $\varphi_{lk}$ .

Table 2

**Sketch of camming contour**

Variant number	Contour sketch	$\varphi_t^0$	$\varphi_{lt}^0$
1		110	10
2		105	15
3		95	20
4		93	17

When solving variants No 2 and No 3, sections of the descent (odd quadrants), where the angles will change in accordance with the dependencies (Table 2) are considered.

2.4.4. Determine the value of the rake  $\gamma_k$  and  $\alpha_k$  clearance kinematic (working) angles of the straight-turning cutter for the case of taking into account the values of the longitudinal feed  $f_l = 0.6$  mm/rev with the following initial data:  $\alpha = 12^\circ$ ;  $\gamma = 5^\circ$ ;  $\lambda = 0^\circ$ ;  $d = 120$  mm. Give a graphic scheme for determining the angles  $\gamma_k$  and  $\alpha_k$ .

2.4.5, Determine the value of the rake  $\gamma_k$  and clearance  $\alpha_k$  kinematic (working) angles of the cutoff tool while taking into account the feed motion  $f_{tr} = 0.6$  mm/rev and the displacement of the cutter corner the relative to the axis of the part is higher by  $h = 3$  mm with the following initial data:  $\alpha = 6^\circ$ ;  $\gamma = -2^\circ$ ;  $\lambda = 0^\circ$ ;  $\varphi = 60^\circ$ ;  $d = 150$  mm.

Give graphic schemes for determining the angles  $\gamma_k$  and  $\alpha_k$ .

2.4.6. Determine the value of the rake  $\gamma_k$  and clearance  $\alpha_k$  kinematic (working) angles of the straight-turning side-facing cutter while simultaneously taking into account the feed motion ( $f_l = 0.8$  mm/rev) and the displacement of the corner of the cutter relative to the axis of the part lower by  $h = 4$  mm with the following initial data:  $\alpha = 6^\circ$ ;  $\gamma = -2^\circ$ ;  $\lambda = 0^\circ$ ;  $\varphi = 90^\circ$ ;  $d = 200$  mm.

Give graphic schemes for determining  $\gamma_k$  and  $\alpha_k$ .

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### 3. PHYSICAL PHENOMENA IN CUTTING

#### *3.1. Brief theoretical information*

Cutting in the modern view in the system plan as a set of simultaneously performed processes: elastoplastic deformations of the cut layer, wear and dulling of the cutting tool, formation of the machined surface of the part is interpreted [1, 5, 23].

The process of elastoplastic deformation (EPD) in three zones: transitional between the cut layer and chips, cutter and restored is carried out (in the area of the machined surface).

In the analytical solution of problems, the most common EPD-model in the transition zone is a model with a single sliding (shearing) plane (experimentally discovered by I.A. Time [24]), which is inclined to the cutting plane at a sliding angle  $\beta_s$  lying in the range of  $15^0...45^0$ . The velocity of chips motion and the velocity of deformation in a single sliding plane  $V_d$  (kinematics of chip formation) depends on the physical and mechanical characteristics of the material being machined, and on the other hand, on the cutting speed  $V$ , geometry the tool cutting part and other factors of the cutting process.

When solving the problems of kinematics of chip formation, it is necessary be base from the model of the cutting process with a single sliding plane and known relations:  $V_c = f(V)$ ;  $V_d = f(V)$ .

In the process of solving the problem using the graphic-analytical method, obtain these two dependencies; connect quantitative data with the physics of the cutting process and, in particular, with force and tool durability interactions in the tool-workpiece contact zone [25-27].

The main quantitative criterion for EPD is the characteristic of chip shrinkage (change in shape and size – shortening length and thickening of chips). The shrinkage factor  $K_L$  depends, first, on the properties of the material being processed, the position of the conditional sliding plane and the geometric parameters of the tool (first, on the rake angle of the tool  $\gamma$ . Analytical definition  $K_L = f(\beta_c, \gamma)$  received I.A. Time [24]. In the process of solving the problem, it is recommended to independently; using the graphical-analytical method, obtain the dependencies  $K_L = f(\beta_c, \gamma)$ .

The degree of chip shrinkage and relative sliding  $\varepsilon$  as a criterion for EPD predetermine, along with adhesion, friction and other factors, force interactions in the cutting process and the work of cutting forces. The work expended on cutting includes the work required for deformation  $E_d$ , the work of friction forces on the face surface  $E_{ff}$  and the work of friction forces on the clearance surface, which is insignificant in magnitude and not taken into account in the calculations [28, 29]. There are full minute  $E$  and specific  $e$  work, referred to a unit volume, cut off in one minute of the cutting process. The force interaction (stress state) in the contact zone of the face surface of the tool and the workpiece is characterized by the angle of action  $\omega$ , measured between the chip formation force  $R$  and the cutting plane.

The connection between the stress and strain state of  $\beta_c = f(\omega)$  was found by the Russian scientist K.A. Zvorykin [30]. Before solving the problems of this section, designer should study all the assumptions put forward by K.A. Zvorykin and obtain the analytical dependence  $K_L = f(\beta_c, \gamma)$  In the process of solving the problem, it is necessary to use knowledge about the systems of forces acting on the contact surfaces of the cutting tool, their ratios and approximate values [31-33].

Thus, this section presents three types of problems related to the kinematics of chip formation, chip settling, force and work of cutting.

### 3.2. Typical task 4. Determine the specific work of deformation

Determine the specific work  $e_d$  of deformation during planning structural steel according to the scheme of free cutting with a single sliding plane.

Given: Main component of cutting force  $P_z = 1020$  N (Fig. 11); friction angle  $\theta = 50^\circ$ ; clearance angle  $\alpha = 10^\circ$ ; wedge angle  $\beta = 70^\circ$ ; cross-sectional dimensions of the cut layer  $a \times b = 2 \times 10$  mm.

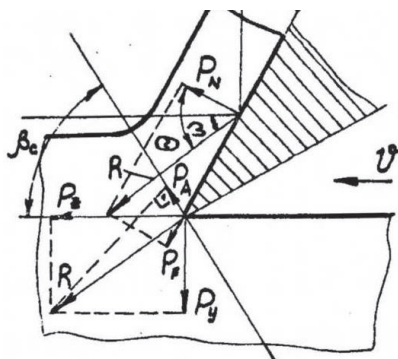


Fig. 11. Scheme of decomposition of cutting forces

*The solution of the problem.* Specific work  $e_d$  as the ratio of the total minute work of the deformation forces directed along the conditional shear plane to the volume  $V_m$  of the cut layer in one minute is defined:  $e_d = e_d / V_m$ , where  $V_m = a \cdot b \cdot V$ , here  $a$ ,  $b$ ,  $V$  – respectively the thickness, the width of the cut layer and the cutting speed. In turn  $e_d$ , it is defined as the product of the deformation force  $P_d$  and the deformation velocity  $V_d$  (coinciding in direction with  $P_d$ ).

Express  $P_d$  through the main component of the cutting force  $P_z$ . To do this, we will draw a diagram of free cutting, on which we will show the system of forces acting on the face surface of the tool [5, 6]. In the process

of cutting, a force of normal pressure  $P_N$  and a friction force  $P_F$  directed in the opposite direction of the chip movement. We transfer the resultant chip formation force  $R$  to the point “0”, coinciding with one of the points of the main cutting edge, and resolution of forces in the sliding plane (component  $P_d$ ) and component force  $P_N$  in the direction perpendicular to it. We find the ratio:  $P_d$  and  $R$ :

$$P_d = R \cdot \cos(\omega + \beta_c).$$

Decompose  $R$  in the cutting plane (component  $P_z$ ) and the reference plane (component  $P_y$ ). We find the ratio  $P_d$  and  $P_z$ :

$$R = \frac{P_z}{\cos \omega};$$

$$P_d = \frac{P_z \cdot \cos(\omega + \beta_c)}{\cos \omega}.$$

We express  $V_d$  in terms of the cutting speed  $V$ . To do this, we drawing cutting pattern of the chip formation kinematics in free cutting (Fig. 12).

Resolution the cutting speed vector into vectors sliding  $V_d$  and friction  $V_F$  speeds on the face surface, using the sine theorem, will have:

$$\frac{V_d}{\sin(90 - \gamma)} = \frac{V}{\sin \delta_c}; \quad \frac{V_d}{\cos \gamma} = \frac{V}{\cos(\beta_c - \gamma)}.$$

Hence the sliding velocity:

$$V_d = \frac{V \cdot \cos \gamma}{\cos(\beta_c - \gamma)}.$$

Specific work of deformation:

$$e_d = \frac{P_z \cdot \cos(\omega + \beta_c) \cdot V \cdot \cos \gamma}{\cos \omega \cdot \cos(\beta_c - \gamma) \cdot a \cdot b \cdot V} = \frac{P_z \cdot \cos(\omega + \beta_c) \cdot \cos \gamma}{\cos \omega \cdot \cos(\beta_c - \gamma) \cdot a \cdot b}.$$

In the above expression,  $\omega$ ,  $\beta_c$  and  $\gamma$  are known in the task. To determine  $\gamma$ , consider the section of the cutter in the major secant plane (Fig. 12) and use the known ratio  $\gamma = 90 - (\alpha + \beta) = 10^\circ$ .

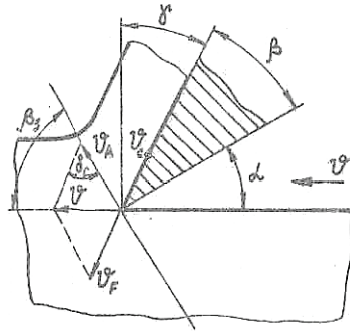


Fig. 12. Cutting speed decomposition scheme

To determine the angle of action  $\omega$ , consider scheme of the resolution of forces (Fig. 12) and connect with the friction angle  $\theta$  ( $\theta$  – the angle measured between  $P_N$  and  $R$ ):  $\omega = \theta - \gamma = 40^\circ$ .

When determining the sliding angle  $\beta_c$ , you need to know about the relationship of the stress state of the cut material (expressed by the angle  $\omega$ ) and the deformed state in the transition zone (expressed by the angle  $\beta_c$ ).

For the first time, this connection was substantiated and analytically obtained a mathematical expression by K.A. Zworykin [30]:

$$\beta_c = \frac{90 - \omega}{2}.$$

Appendix 3 contains the main assumptions and the complete derivation of the Zworykin formula. Using this formula, we find the sliding angle  $\beta_c = 25^\circ$ . Let's define a numerical value specific work of deformation  $e_d$ :

$$e_d = \frac{1020 \cdot \cos 60^0 \cdot \cos 10^0}{2 \cdot 10 \cdot \cos 40^0 \cdot \cos 15^0} = 28.69, \text{ N/mm}^2.$$

### 3.3. Tasks for independent solution

3.3.1. Determine the angle of inclination of the conditional sliding plane  $\beta_c$ , if in the zone of contact between the chip and the face surface of the tool [5].

Given at the average normal stress  $\sigma_N = 400 \text{ MPa}$ ; the average tangential  $\tau_F = 300 \text{ MPa}$ ; clearance angle  $\alpha = 12^0$ ; wedge angle  $\beta = 70^0$ ; processed material – steel 5120 Standard AISI (steel 20X).

When deciding, derive the Zworykin formula and give a graphical diagram for determining the forces acting on the face surface of the tool.

3.3.2. Determine the value of the strain rate  $V_d$  and chip rate  $V_{ch}$  at the following values: cutting velocity  $V = 72 \text{ m/min}$ ;  $\beta_c = 30^0$ ;  $\delta = 85^0$ .

When solving, it is necessary to derive the dependencies  $V_d = f(V)$  and graphically drawing the kinematics of chip formation in free cutting.

3.3.3. Determine the chip shrinkage coefficient  $K_L$  and the relative shearing  $\varepsilon$  when planning structural steel (consider the case of free cutting with a single sliding plane).

Given: clearance angle  $\alpha = 10^0$ ; wedge angle  $\beta = 70^0$ ; friction angle  $\theta = 50^0$ .

Solution. Output dependencies  $K_L = f(\beta_c, \gamma)$ ;  $\varepsilon = f(\beta_c, \gamma)$  and graphically drawing diagrams illustrating shrinkage and relative shearing, diagrams of the action of forces on the face surface of the cutter and the section of the cutting wedge in the corresponding coordinate plane.

3.3.4. According to the requirements of the drawing, a certain group of parts must be made from a new brand of structural material. To get practical calculations, you need:

1. Define a particular empirical dependence of the cutting force component  $P_z$  on the depth of cut  $d_c$ . The experimental data are shown in Table 3. Solve the problem by the least squares method.

2. Define generalized dependence  $P_z$  on the feed ( $f_c$ ) and depth of cut ( $d_c$ ), provided that the particular dependence  $P_z$  on the  $f_c$  has the form:

$$P_z = 3370.0 \cdot f_c^{0.66}.$$

The dependence was obtained at  $d_c = 5$  mm.

Table 3

**Experimental data**

$d_c$ , mm	3	5	7	10	Note
$P_z$ , N	1490	2380	3500	4700	$f = 0.6$ , mm/rev

3.3.5. Determine the specific work of the deformation forces  $e_d$  in the scheme of free cutting with a single shearing plane.

Given: cutting force component  $P_z = 1500.0$ , N; angle of action  $\omega = 20^\circ$ ; tool clearance angle  $\alpha = 12^\circ$ ; wedge angle  $\beta = 70^\circ$ . Chip width  $b = 5$  mm; shearing plane length  $L_s = 3$  mm.

When solving, an analytical conclusion  $e_d$  and a graphic representation of the system of forces acting on the face surface of the tool and the section of the cutter in the corresponding plane are obligatory.

3.3.6. Determine the specific work of friction forces on the face surface of the tool  $e_f$ .

Given:  $P_z = 2000.0$ , N; friction angle  $\theta = 45^\circ$ ; tool cutting angle  $\delta = 80^\circ$ ; chip width  $b = 10$  mm; shearing plane length  $L_s = 3$  mm.

When solving, it is necessary to derivation an analytical formula  $e_f$  and give a graphical drawing of the forces system acting on the tool face surface and the section of the cutter in the corresponding plane.

3.3.7. Determine the specific work of friction  $e_F$  on the tool face surface with the following initial data: the main tangential component of the cutting force  $P_z = 3000$ , N; the thickness and width of the cut layer are respectively equal  $a = 1.0$ , mm and  $b = 15.0$  mm; chip shrinkage factor  $K_L = 2.0$ ; friction angle  $\theta = 50^\circ$ ; cutting angle  $\delta = 80^\circ$ .

When solving, the analytical derivation  $e_F$  and a graphic representation of the system of forces acting on the tool face surface and the section of the cutter in the corresponding plane [6].

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## 4. HEAT AND TOOL LIFE PHENOMENA IN CUTTING

### *4.1. Brief theoretical information*

Most of the cutting work is converted into heat. In the process of studying it is necessary, pay special attention to a number of competitive phenomena (for example, associated with an increase in the rate of deformation and a decrease in the cutting force with an increase in the cutting speed), leading to a change in temperature in the cutting zone [3, 4]. Use these phenomena when explaining the dependence of the cutting temperature on the elements of the cutting mode, the characteristics of the processed and tool materials, and the geometry of the tool. Tasks for determining the amount of heat released in the three main deformation zones are associated with the calculation of the work released in this zone, one of which we considered (typical Task 3).

In the process of cutting, the dimensions, shape and crystal structure of the cutting tools material are changed. When studying tool wear, it is necessary to learn the main phenomena leading to wear (abrasive action, adhesion, diffusion, thermal phenomena and oxidation of surface layers), and external appearance of wear (facet on the flank and crater on the face surfaces, etc.) [1, 34-36]. Considering the intensity of wear as the rate of growth of the wear mass, pay attention to the dependence of the nature of its change on the cutting speed:

- extreme (when machining steel); monotonically increasing (when machining cast iron with single-carbide hard alloys W-Co type);
- extreme with a minimum value in the speed range  $V = 200 \dots 300$  m/min (when machining cast iron with two-carbide hard alloys Ti-W-Co).

When assigning a dulling criterion, it is important to know the areas of effective application of the criteria of equal, optimal and technological dulling. At the same time, based on information about the actual processing conditions (roughing, finishing) assign one or another criterion.

Wear curves of the tool (a graphical representation of the change for wear over time) serve as the basis for constructing Tool Life dependencies in the coordinates "tool life-cutting speed",  $T-V$ .

In the process of studying the discipline of Cutting Theory, one should learn how to build tool life distribution curves by the experimental method, find sections  $T = f(V)$  that correspond to optimal working conditions, choose such values of  $V$  that deliver the maximum value of tool life. The tasks for determining the wear optimality criterion are based on the method of N.N. Zorev [37], who analytically obtained the expression for the dependence  $h_c = f(T)$  – height of the wear area  $h_c$  on the flank surface of the tool from tool life [2]. This dependence applies to both regrindable and non-regrindable cutting inserts (the number of non-regrindable edges is equal to the number of regrinds).

However, given the statistical data, using the characteristic features of the wear curve, it is possible to determine the transition points from the wear-in section to the normal wear section and from the latter to the forced wear section. The value  $h_c$ , corresponding to the transition to forced wear, is usually associated with optimal wear.

The dependence  $m = f(T)$  more objectively reflects the physical phenomena underlying the wear process of the tool material along the entire cutting length of the blade, while the dependence  $h_c = f(T)$  shows the pattern of increasing local wear at one point of the blade. Calculations show that on the curve  $m = f(T)$  there are no inflection points – there is a monotonous increase in the wear mass  $m$  on throughout the cutting time of this cutter.

The problems of determining tool wear and tool life are associated with the use of the tool life dependence  $T = f(V)$ , which has a complex multi-extremal character. In real production conditions, under which processing is carried out at sufficiently high speeds, to describe the resistance dependence, power functions are used in the form:

$$T = \frac{C_T}{V^{1/\mu}},$$

where  $1/\mu = m_T$  – indicator of relative tool life;  $C_T = C_V^{1/m_T}$  is a constant characterizing the influence of the parameters of the tool and work materials, the geometry of the cutting part of the tool and other cutting conditions. Numeric values  $m_T$  and  $C_V$  are determined from reference books.

In tasks where relationships between different cutting speeds and tool life are determined, it is necessary to use the dependencies  $V_1 \cdot T_1^{m_T} = V_2 \cdot T_2^{m_T} = \dots = V_n \cdot T_n^{m_T} = \text{const.}$

#### **4.2. Typical task 5. Determine the optimal period of tool life**

In the machine shop, under conditions of mass production, a turning operation is performed with expensive cutters with soldered tips (insert) made of DIN HS021 (T30K4, GOST 3882-74) hard alloy.

It is required to determine the optimal period of tool life  $T$  using the criterion of optimal wear (method of N.N. Zorev) according to the data given in Table. 4; build dependency graphs  $h_c = f(T)$  and  $m = f(T)$ .

Table 4

**Initial data**

Experience number	1	2	3	4	5	6	7	8	9
$h_c$ , mm	0.05	0.1	0.11	0.36	0.47	0.58	0.68	0.8	0.95
$T$ , min	1	2	3	25	50	75	100	110	115

Footnote. It is allowed not to build a cutter wear-in section.

Geometrical parameters of the cutting insert:  $\alpha = 8^\circ$ ;  $\gamma = 0^\circ$ , total length of the cutter tip  $l = 20$  mm, allowable amount of overcast  $M = \frac{2}{3}l$ , the width of the wear part of the cutter along the major clearance surface of tool  $b = 15$  mm.

*The solution of the problem.* The criterion of optimal wear according to N.N. Zorev [37], involves determining such a wear height  $h_c$  along the flank facet, which would provide maximum total tool life  $T_\Sigma$ .

Using experimental data (Table 4), we define  $T_\Sigma$  in the form of a dependency:  $T_\Sigma = \Sigma T(i+1)$ , where  $T$  – period of tool life between two sharpening;  $i$  – number of regrinding allowed by the tool to its full depreciation. To determine the optimal wear  $h_c^{opt}$ , we construct a wear curve according to Table 4. Analysis of changes in these data (Fig.13) shows that, starting from  $T = 100$  min, there is a sharp increase that is transition to the area of forced wear  $F$ ., This inflection point is the value of optimal wear  $h_c^{opt}$  taking the value  $h_c^{opt} = 0.68$  mm.

This value  $h_c^{opt}$  corresponds to the tool life period  $T = 100$  min

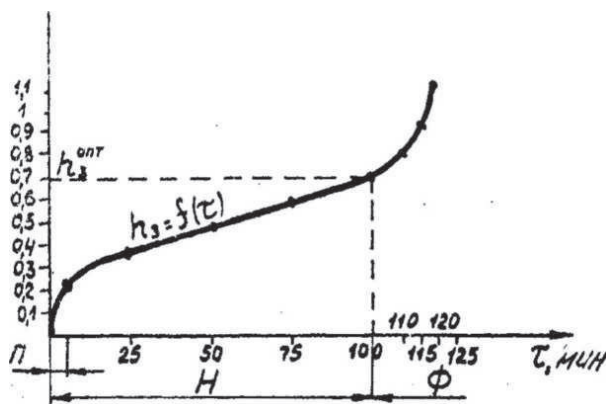


Fig. 13. Cutter wear optimal

To find the number of regrinds  $i$ , we will drawing the tool regrinding scheme (Fig. 14). In order for the tool to become operational, from the flank surface during regrinding, a layer of hard alloy (HA) with a thickness of  $h + \Delta h$ , must be ground off. Here  $\Delta h = 0.1 \dots 0.15$  mm, includes a sharpening tolerance and a layer of HA ground to remove defects under the wear part of the flank surface [38-40]. Permissible number of sharpening

$i = \frac{H}{h + \Delta h}$ , while  $h$  is measured perpendicular to the reference surface of the tool.

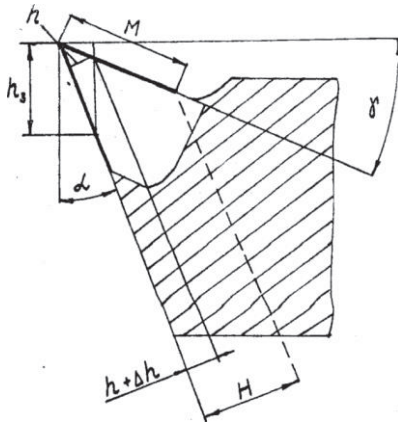


Fig. 14. Scheme of tool regrinding

Based on trigonometric transformations:

$$h = h_c \cdot \operatorname{tg} \alpha \cdot \frac{\cos(\alpha + \gamma)}{\cos \gamma}.$$

Let's express the value  $H$  in terms of the permissible value of the overcast  $M$ :

$$H = M \cdot \cos(\alpha + \gamma).$$

The total tool life period equals:

$$T_{\Sigma} = \frac{H \cdot T}{h + \Delta h} + T = \frac{M \cdot \cos(\alpha + \gamma) \cdot T}{\left( h_c \cdot \operatorname{tg} \alpha \cdot \frac{\cos(\alpha + \gamma)}{\cos \gamma} \right) + \Delta h} + T =$$

$$= \frac{2 \cdot 20 \cdot \cos 8^\circ \cdot 100}{3(0.68 \cdot \operatorname{tg} 8^\circ \cdot \cos 8^\circ) + 0.1} + 100 = 6873.4 \text{ min.}$$

The dependence of the mass of the tool wear part along the flank surface on  $T$  is determined from the ratio obtained as a result of trigonometric transformations (Fig. 14):

$$m = \frac{b \cdot h_c^2 \cdot \rho \cdot \operatorname{tg} \alpha \cdot \cos(\alpha + \gamma)}{2 \cdot \cos \gamma \cdot \cos \alpha}.$$

This ratio was obtained analytically based on the assumption of constancy  $h_c$  along the major blade of the tool (Fig. 15) [41-43]. Compile a calculation table (Table 5) to determine  $m = f(T)$ .

Table 5

Initial data									
Option number	1	2	3	4	5	6	7	8	9
$h_c$ , mm	0.05	0.1	0.11	0.38	0.47	0.58	0.68	0.8	0.95
$m$ , N	0.005	0.01	0.0011	0.0048	0.0049	0.05	0.07	0.032	0.0098
$T$ , min	1	2	3	25	50	75	100	110	115

When determining the mass, we take the density of the hard alloy DIN HS021 (T30K4)  $\rho = 9800 \text{ kg/m}^3$ . In Fig. 15 conventionally does not show the running-in section.

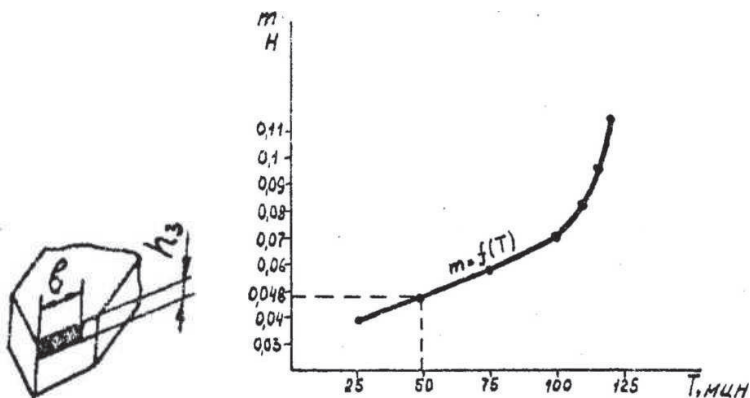


Fig. 15. Diagram of mass change upon wear

Based on the calculations we have obtained the optimal total tool life period is 6873 min. Dependences of the height of the wear facet and the wear mass over time are shown in Fig. 13 and Fig. 15.

### 4.3. Tasks for independent solution

4.3.1. In a mass production enterprise, a large overspread hard alloy. One of the solutions to reduce consumption is to find a criterion for optimal wear. What is its essence? The wear of a straight-turning cutter with a P10 ISO 513 (T15K6) cutting tip when processing a part made of steel DIN 41 Cr 4 (40X) based on the normative. With diameter:  $D = 60$  mm;  $l = 200$  mm; depth of cut  $d_c = 1.5$  mm;  $f = 0.15$  mm/rev to construct for the tool life period  $T$  its characteristic wear curve. Find the criterion for optimal wear if the shape of the tip is № 0227. How many parts can be machined with a cutter with the found criterion for optimal wear? Compare it with the standard.

4.3.2. Determine the volume of the wear mass on the flank surface of the cutter, if the height of the wear facet is  $h_c = 3$  mm; wear facet width  $b = 12$  mm; tool clearance angle  $\alpha = 10^\circ$ ; wedge angle  $\beta = 70^\circ$ .

4.3.3. Determine the amount of heat  $Q_d$ , released for 2 min in the deformation zone by free cutting with a single shearing (sliding) plane.

Given:  $P_z = 900$  N; tangential stresses in the contact zone (rake surface of the tool and chips)  $\tau_F = 300$  MPa; normal stresses in the contact zone  $\sigma_N = 300$  MPa; tool clearance angle  $\alpha = 10^\circ$ ; wedge angle  $\beta = 70^\circ$ ; cutting speed  $V = 20$  m/min.

4.3.4. Determine the amount of heat  $Q_d$ , released for 10 min in the deformation zone (in the scheme of free cutting with a single shearing plane).

Given:  $P_z = 1080$  N; tangential stresses in the contact zone  $\tau_F = 300$  MPa; normal stresses in the contact zone  $\sigma_N = 340$  MPa; tool clearance angle  $\alpha = 12^\circ$ ; wedge angle  $\beta = 80^\circ$ ; cutting speed  $V = 20$  m/min. Perform all the necessary analytical transformations to determine the forces acting on the rake surface of the tool, and drawing the cutter section in the corresponding coordinate plane.

4.3.5. Determine the amount of heat  $Q_F$  released in 5 min in the friction zone of the chip and the rake surface of the tool.

Given:  $P_z = 980$  N; tangential stresses in the contact zone  $\tau_F = 320$  MPa; normal stresses in the contact zone  $\sigma_N = 300$  MPa; cutting angle  $\delta = 75^\circ$ ; cutting speed  $V = 15$  m/min.

Give all the necessary analytical transformations, diagrams of the forces acting on the rake surface of the tool, and the section of the cutter in the corresponding coordinate plane.

4.3.6. Determine the calculated height of microroughnesses  $R_{zc}$  at processing with a sharpened cutter.

Given: the major angle in the plan  $\varphi = 45^\circ$ ; angle at corner of cutter in plan view  $\varepsilon = 120^\circ$ ; the thickness of the cutting layer  $a = 1.2$  mm. When solving, derivation the dependencies  $R_{zc} = f(f_c, \varphi, \varphi_1, r)$  and give graphical schemes for determining  $R_{zc}$ .

4.3.7. Determine the calculated height of microroughnesses  $R_{zc}$  with a radius at the cutter corner  $r_c = 1.2$  mm; thickness of the cutting layer  $a = 2.0$  mm; major angle in plan  $\varphi = 60^\circ$ . Provide a graphical representation of the calculated microprofile. When solving, it is necessary to derivation dependencies  $R_{zc} = f(f_c, \varphi, \varphi_1, r)$  give graphical schemes for determining  $R_{zc}$ .

4.3.8. When processing plastic metals, the temperature of the rake surface of the cutter:

- 1 – higher than the temperature of its clearance surface?
- 2 – below the temperature of its clearance surface?
- 3 – equal to the clearance surface temperature?

4.3.9. With an increase in the major angle in plan - heat removal from the cutting edge:

- 1 – will not change?
- 2 – getting better?
- 3 – getting worse?

4.3.10. When machining with carbide tools, the exponents at cutting speed, feed and depth of cut:

- 1 – less than when working with high-speed cutters?
- 2 – more than when working with high-speed cutters?
- 3 – equal to the indicators when working with high-speed cutters?

4.3.11. Under equal operating conditions, in which of the following cases will the cutting temperature be higher:

- 1 – when the tool is equipped with P10 ISO 513 (T15K6) hard alloy?
- 2 – K20-K30 (VK6)?
- 3 – DIN HS 021 (T30K4)?

4.3.12. Under the same working conditions, in which case the cutting temperature will be higher:

- 1 – when is the tool equipped with a carbide?
- 2 – when the tool is equipped with mineral ceramics?

4.3.13. Under equal working conditions, the temperature of the cut will be higher when:

1 – is the tool equipped with K20-K30 (VK6)?

2 – is the tool equipped with P30-P40 (T5K10)?

4.3.14. Under equal operating conditions, the cutting temperature will be higher when:

1 – the instrument equipped with white mineral ceramics?

2 – the tool equipped with black mineral ceramic (cermet)?

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## 5. CALCULATION OF CUTTING CONDITIONS USING REFERENCE LITERATURE

### *5.1. Brief theoretical information*

The definition of the cutting mode consists in choosing, according to the given processing conditions (technical requirements for the accuracy of the machined surface, the design of the cutting tool and the material of its cutting part). In addition, the decision-making process takes into account: the physical and mechanical properties of the material being machined, the permissible wear of the tool, its tool life and geometric parameters of the cutting part. As a result, it is necessary to obtain the most advantageous combination of cutting depth, feed and cutting speed, providing the lowest labor intensity and cost of the operation [44-46].

Cutting conditions are set in the following order:

1) determination of the depth of cut  $d_c$  (mm) and the number of passes. Depending on the type of processing and the requirements for surface quality, a variant of partitioning the allowance on the surface to be machined along the passes is selected. When rough turning, it is advisable to remove the entire allowance in one pass. With roughing passes with a long machine time, when the establishment of higher cutting conditions is limited by the power of the machine, in some cases it is more profitable to remove the start-up in several passes. The feasibility of this should determine by comparative calculation of the duration of the operational time required to perform a given technological operation with a different number of passes [47-49]. The division of allowances into several passes is also carried out during semi finishing and finishing turning of non-rigid

parts, as well as during processing with cutters with an additional cutting edge,  $\varphi_1 = 0$ ;

2) feed selection  $f_c$ , mm/rev. Feeds according to the reference literature depending on the cross-sectional square of the cutter holder; the processing diameter and the depth of cut are selected;

3) determination of the standard cutting speed  $V$ , m/min and the corresponding rotational frequency  $n$ ,  $\text{min}^{-1}$ , permissible for a given period of cutting tool life. In practice, there are two options for determining – according to maps [46, 50, 51] (depending on the depth of cut, feed and major angle in plan of the tool) or according to the well-known empirical formula [45]. The average value of tool life for single-tool turning is 30...60 min, and for drilling it is selected according to the table, depending on the material being processed and tool materials, taking into account the diameter of the drill [45, 52, 53]. According to the speed value, the required spindle speed is selected and according to the machine passport is corrected;

4) determination of cutting force  $P$  and cutting power  $N$  according to the selected depth – cutting, feed and cutting speed. These parameters can be found either according to existing normative cards [44, 46], or with the help of empirical formulas [45];

5) check the possibility of implementing the selected cutting mode on a given machine tool according to the operational data [54-56]. If the found cutting mode cannot be carried out on a given machine, (effective power required for cutting is higher than the power on the spindle) and the selected feed satisfies the above constraints, it is necessary to reduce the cutting speed. The reduction of speed value  $V$  by entering the correction factor  $K_V$  for speed change is carried out [46]. It is depending on the ratio of the power on the spindle, allowed by the machine to the power according to the standards;

6) correction of the selected mode for the machine in accordance with its passport (certificate) data.

The given sequence of calculation during drilling, milling and other types of blade cutting is mainly preserved.

### ***5.2. Typical tasks. Calculate cutting modes during turning***

Calculate cutting modes during preliminary turning of a shaft-type part on a 16K20 machine.

Initial data; type and size of the workpiece – rolled steel DIN C 45 (steel45); ultimate tensile strength ( $\sigma_u = 550 \text{ MPa}$ );  $D = 80 \text{ mm}$ ;  $m = 19 \text{ kg}$ ; tool – straight-turning cutter, equipped with a tip of hard alloy P30 (T5K10). Cutter geometric parameters:  $\varphi = 45^\circ$ ;  $\varphi_1 = 10^\circ$ ; cutter tip thickness  $c = 4 \text{ mm}$ ; tool holder dimensions  $B \times H = 25 \times 25 \text{ mm}$ ; cutter overhang  $l_c = 1.5 \cdot H$ . Conditions for performing the operation: workpiece – mounted into a self-centering three-jaw chuck with the center of the tailstock being compressed; surface treatment I (Fig.17) is performed in one pass.

The calculation of cutting conditions in the traditional sequence using the data from [46] is performed.

1. Let's drawing the shaft-processing scheme (Fig. 16).
2. Let's determine the depth of cut, which is taken equal to the machining allowance in one pass:

$$f_c = (D - d) / 2 = (80 - 68) / 2 = 6 \text{ mm}.$$

3. We select the feed  $f_c$ : a) using card 4 [46], we determine the feed value for rough turning. For a tool holder with a section of 25x25 mm; machining diameter up to 100 mm and depth of cut up to 8 mm – the recommended feed:  $f_c = 0.5 \dots 0.7 \text{ mm/rev}$ .

Let's check the admissibility of supplying the recommended card 4 according to the power of the electric motor (card 13), the strength of the cutter holder (card 14) and the strength of the hard alloy tip (card 15).

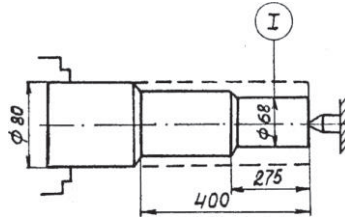


Fig. 16. Shaft processing scheme

According to card 13 [46], for a depth of cut  $d_c = 6$  mm, electric motor power  $N_m = 8$  kW, and for a cutter  $\varphi_1 > 0^\circ$ , a feed  $f_c = 0.7$  mm/rev. is allowed. In the same card for steel with tensile strength  $\sigma_u = 550$  MPa we find the correction factor  $K_{mf} = 1.07$ . Consequently, the feed allowed by the power of the electric motor (from the conditions for ensuring work for a hard alloy at a speed of at least 50 m/min,  $f_c = 0.7 \cdot 1.07 = 0.75$  m/rev.

On card 14 [46] for a cutter with a holder with a cross section of  $25 \times 25$  mm and a depth of cut  $d_c = 6$  mm, we find the feed  $f_c = 3$  mm/rev. By multiplying this feed rate by the correction factor corresponding  $K_{mf}$  to the steel with ultimate strength  $\sigma_u = 550$  MPa, corresponding to the tool overhang  $l_c = 1.5 \cdot H$ , we find the feed rate allowed by the tool holder tensile strength:  $f_c > 3 \cdot 1.7 \cdot 0.58 = 1.86$  mm/rev.

According to card 15 [46] for a cutter with a major angle in plan  $\varphi = 45^\circ$ , a thickness of a hard alloy tip  $c = 4.0$  mm and for a depth of cut  $d_c = 6$  mm we find the feed  $f = 1.11$  mm/rev. Taking into account the correction factor for steel:  $\sigma_u = 550$  MPa,  $K_{mf} = 1.07$ , allowable feed  $f_c = 1.11 \cdot 1.07 = 1.19$  mm/rev.

From the comparison of feeds determined by the cards 13:  $f_c = 0.7$  mm/rev, card 14:  $f_c > 1.86$  mm/rev and 15:  $f_c = 1.19$  mm/rev,

we see that the feed rate limits the power of the electric motor. But the feed allowed by the power of the electric motor does not limit the maximum feed recommended by card 13, i.e. feed  $f_c = 0.7$  mm/rev. Such a feed is available on the machine tool (according to the passport data), therefore, we will accept it to perform the technological operation of processing for a shaft section of diameter  $\varnothing 68$  mm.

4. Select the cutting speed and rotation frequency of spindle. By card 19 (sheet 1) by cutting depth of cut  $d_c = 6$  mm for straight turning cutter with a major angle in plan  $\varphi = 45^\circ$  for  $f_c = 0.7$  mm/rev finds:  $V = 100$  m/min;  $P_z = 6630$  N;  $N_e = 10.7$  kW.

Using card 45, we determine the correction factors for the changed cutting conditions. In this example, it is necessary to take into account only the correction-cutting coefficient depending on the tensile strength of the processed material  $\sigma_u$ . For  $\sigma_u = 550$  MPa on card 45 (sheet 2) we find  $K_{mV} = 1.18$ ;  $K_{mp_z} = 0.92$ ;  $K_{mN_e} = 1.09$ .

Therefore, for given processing conditions, the standard values  $V$ ,  $P_z$  and  $N_e$  are:

$$V = 100 \cdot 1.18 = 118 \text{ m/min}; P_z = 6630 \cdot 0.92 = 6100 \text{ N};$$

$$N_e = 10.7 \cdot 1.09 = 11.6 \text{ kW}.$$

The found mode on a given machine, since the effective power required for cutting is  $N_e = 11.6$  kW higher than the power on the spindle, the permissible rated power of the electric engine  $N_e = 11$  kW according to the passport of the machine cannot be implemented. Need to slow down cutting speed. The coefficient of change in cutting speed depends on the ratio power on the spindle, allowed by the machine, to the power according to the standards.

In this example, this ratio will be  $11/11.6 = 0.94$ . According to the card 45 (sheet 7) [46] for the ratio we find the rate of change of speed  $K_v = 0.88$ . Cutting speed, set according to the power of the machine:

$V = 11.8 \cdot 0.88 = 103.8$  m/min. Spindle rotation frequency  
 $n = \frac{1000 \cdot V}{\pi \cdot d} = \frac{1000 \cdot 103.8}{3.14 \cdot 80} = 413$  rpm. According to the passport of the machine we choose 400 rpm.

Finally, for machining operation of shaft section  $\varnothing 80$  mm: feed  $f_c = 0.7$  mm/rev and rotation frequency  $n = 400$  rpm. Actual cutting speed:  $V = \frac{\pi \cdot d \cdot n}{1000} = 100.5$  m/min.

In the practice of technological calculations, another approach to the calculation of cutting conditions, based on the use of empirical formulas for determining the cutting speed, the components of the cutting forces and cutting power. For this task, let us define these components of the cutting mode is also used.

The cutting speed by the empirical formula is determined:

$$V = \frac{C_v \cdot K_v}{T^{m_v} d_c^{x_v} f_c^{y_v}}.$$

Coefficient  $K_v$  is the product of coefficients that take into account the influence of the workpiece material  $K_{m_v}$ , surface condition  $K_{c_v}$ , tool material  $K_{t_v}$  on the cutting speed. The values of the coefficient  $C_v$  and the exponents  $x_v$ ,  $y_v$ ,  $m_v$  are given in [45, Table 17].

For this task of calculating cutting modes, we will take [45]:

$$K_{m_v} = K_r \cdot \left( \frac{750}{\sigma_u} \right)^{n_v} = 1.0 \cdot \left( \frac{750}{550} \right)^1 = 1.36; K_{c_v} = 0.9; K_{t_v} = 0.65; K_v = 0.8;$$

$$C_v = 350; x_v = 0.15; y_v = 0.35; m_v = 0.2.$$

The average value of tool life during processing in the range of 30...60 min is recommended to be taken. Let's take  $T = 60$  min.

Calculate the cutting speed:

$$V = \frac{350 \cdot 0.8}{60^{0.2} \cdot 6^{0.15} \cdot 0.7^{0.35}} = 106.68 \text{ m/min.}$$

Determine the spindle rotational frequency:

$$n = \frac{1000 \cdot V}{\pi \cdot d} = \frac{1000 \cdot 106.68}{\pi \cdot 80} = 424.47.$$

We accept according to the passport of the machine  $n = 400$  rpm.  
Adjusted speed value:  $V = \pi \cdot d \cdot n / 1000 = 100.5$  m/min.

The cutting force  $P$  during longitudinal turning into three components of the cutting force:  $P_z$  – major (tangential);  $P_y$  – radial;  $P_x$  – axial is decomposed. These components are calculated by the formula:

$$P_{z,y,x} = 10 \cdot C_p \cdot d_c^{x_p} \cdot f_c^{y_p} \cdot V^{n_p} K_p.$$

Let us determine the value of the factors of this dependence for  $P_z$ :

$$C_{p_z} = 300; x_p = 1.0; y_p = 0.75; n_p = -0.15;$$

$$K_{p_z} = K_{m_p} \cdot K_{\phi_p} \cdot K_{\gamma_p} \cdot K_{\lambda_p} \cdot K_{r_p} = 0.79 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 0.79;$$

$$P_z = 10 \cdot 300 \cdot 6^1 \cdot 0.7^{0.75} \cdot 100.58^{-0.15} \cdot 0.79 = 5434.73 \text{ N.}$$

Cutting power takes on the meaning:

$$N = \frac{P_z \cdot V}{1020 \cdot 60} = \frac{5434.73 \cdot 100.5}{1020 \cdot 60} = 8.92 \text{ kW.}$$

Finally for machining operation of shaft section  $\varnothing 80$  mm:

$$f_c = 0.7 \text{ mm/rev; } n = 400 \text{ min}^{-1}; V = 100.5 \text{ m/min; } P_z = 5434.73 \text{ N;}$$

$$N = 8.92 \text{ kW.}$$

Along with the traditional tasks of determining the cutting modes in the practice of mechanical engineers, there are tasks that are directly related to the technique of choosing the elements of the cutting mode for other types of processing (drilling, milling) [57-59].

### 5.3. Typical task 7. Calculate cutting modes during drilling

Eliminate the risk of cutaway out the workpiece from gray cast iron HB = 260 when through-hole drilling (drill with a cutting part made of high-speed steel DIN 1.3343 (R6M5)). The drill diameter  $d_t = 16$  mm according to GOST 10903-77 with a standard sharpening of  $\varphi = 116^\circ$  due to the reduction of the axial component  $P_a$  of the cutting force by 25...30%. By how much is it allowed to increase the cutting speed so that the tool life decreases by no more than two times. Cutting mode:  $V = 17.3$  m/min;  $f_t = 0.4$  mm/rev; passage length  $l = 32$  mm.

The correction factor for the axial component is given below Table 6:

Table 6

**Correction factor  $K_{\varphi P}$**

$2\varphi^0$	116...118	120...130	100...115
$K_{\varphi P}$	1.32	1.32	0.87

The solution of the problem. Let's drawing the scheme of through-hole drilling (fragment of the scheme, Fig.17). Let us determine the factors that make it possible to exclude the danger of a cutaway. Dependence of axial force  $P_a$  on various factors [45]:

$$P_a = 10 \cdot C_{P_a} \cdot d_t^{q_P} \cdot f_t^{y_P} \cdot K_{P_a} \cdot K_{\varphi_P}.$$

1. Find the value of the axial component of the cutting force  $P_a$ , at which cutaway is possible. From Tables 9 and 32 [45] we choose component values (1):

$$C_{P_a} = 42.7; q_P = 1.0; y_P = 0.8; K_{P_a} = K_{mP} = \left( \frac{HB}{190} \right)^{0.6} = 1.21.$$

In this case, the axial drilling force  $P_a$  (1) takes on a value:

$$P_a = 42.7 \cdot 16 \cdot 0.4^{0.8} \cdot 1.21 \cdot 1.32 = 524.27 \text{ N.} \quad (1)$$

2. Determine the period of tool life  $T$  (2), which can cutaway:

$$T = \frac{(C_v \cdot K_v)^{\frac{1}{m}} \cdot d_t^{\frac{q_p}{m}}}{V^{\frac{1}{m}} f_t^{\frac{y_p}{m}}} = \frac{(17.1 \cdot 0.67)^8 \cdot 16^2}{17.3^8 \cdot 0.4^{3.2}} = 189.4 \text{ min.} \quad (2)$$

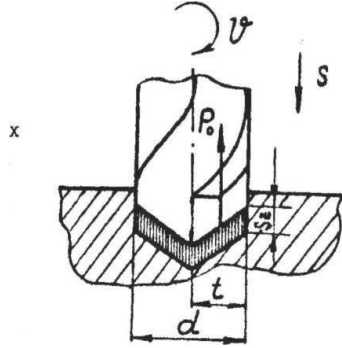


Fig. 17. Drilling scheme

When calculating, it is necessary to use the data presented in [45]:  $C_v = 17.1$ ;  $q_v = 0.25$ ;  $y_v = 0.4$ ;  $m = 0.125$  (at  $f_t > 0.3$  mm/rev).

3. Let us determine the values of the axial component  $P_a$  and period of tool life  $T$ , at which the danger of cutaway is excluded; at the same time, a decrease in tool life is permissible by no more than 2 times. Let's make a system of equations (3):

$$\begin{cases} 0.7 \cdot P_a = K_1 \cdot f_t^{0.8}; \\ 0.5 \cdot T = \frac{K_2}{V^8 \cdot f_t^{4.4}}, \end{cases} \quad (3)$$

where  $K_1$  – the value of the factor  $(10 \cdot C_{p_a} \cdot d_t^{q_p} \cdot K_p \cdot K_{\phi_p})$  from the expression (1);  $K_2$  – the value of the factor  $(C_v \cdot K_v)^{\frac{1}{m}} \cdot d_t^{\frac{q_p}{m}}$ .

Analysis of the system (3) shows that the decrease in the axial component is possible only by reducing the feed [4, 60, 61]. Calculate the value  $f_t$ , at which the axial component decreases by 30%:

$$f_t^{0.8} = \frac{0.7 \cdot P_a}{1091.21}; \quad f_t = 0.26 \text{ mm/rev.}$$

Then, in order for the tool life  $T$  to decrease by no more than 2 times, the cutting speed can be increased to a value  $V = 23.4 \text{ m/min.}$ :

This value is determined from the equation:

$$94.72 = \frac{(14.7 \cdot 0.67)^8 \cdot 16^2}{V^8 \cdot 0.26^{4.4}}; \quad V = 23.4 \text{ m/min.}$$

Here it is important to pay attention to the specifics of working with a reference table (for example, Table 28 [45]). When reducing the feed rate to  $f_t = 0.26 \text{ mm/rev} < 0.3 \text{ mm/rev}$  the value of the coefficient  $C_v$ ,  $K_v$  and the degree index  $q$ ,  $y$ ,  $m$  should be selected from the line in which  $f_t \leq 0.3 \text{ mm/rev}$ , while when determining the initial variant, the reference data were selected from the line with  $f_t > 0.3 \text{ mm/rev}$ .

As a result of solving this problem we can conclude that for eliminate the danger of cutaway, the feed should be reduced to  $f_t = 0.26 \text{ mm/rev}$ . In this case, the axial component of the cutting force  $P_a$  will decrease by 30% and will be  $P_a = 524.27 \text{ N}$ . With this decrease in feed  $f_t$ , it is allowed to increase the speed to the value  $V = 23.4 \text{ m/min}$ . This in turn, will lead to a decrease in the tool life  $T$  in two times up to  $T = 94.72 \text{ min}$ .

### ***5.4. Tasks for independent solution***

5.4.1. During rough turning of shaft on an automatic line, the tool life  $T_1$  at cutting speed  $V_1$  does not suit production due to hourly line stops for

tool replacement. Required to increase tool life by 1.5 times. How does this change the cutting speed? Define  $V_1$  and  $V_2$ .

Shafts made of steel  $\varnothing 40$  mm of steel DIN C 50 E (steel 50), HB = 220 are machined with cutters from P 10 ISO 513 (T15K6).

5.4.2. Find the best option for machining a part made of cast iron HB = 150,  $\varnothing 75$  mm, length  $l = 400$  mm with hard alloy K30-K40 ISO513 (VK8) cutter on a 16K20 machine, depth cutting  $d_c = 4.5$  mm. Limiting Factors: 1. Roughness  $R_z = 40 \mu\text{m}$ ; 2. Tool life; 3. Power.

In order to solve this problem it is necessary to:

1. Calculate the feed allowed by the tool life and power at the same time.

2. Show graphically the region of admissible values of the cutting modes and its optimal values.

5.4.3. There was a production need for the processing of external turning hardened steel parts of the shaft type (HRC 50...52), on the side surface of which, in addition, there is a longitudinal groove 10 mm deep and 10 mm wide. Turning must be done with a straight-turning cutter with a carbide tip (GOST 18878-730),  $B \times H = 16 \times 25$  mm and 6 mm tip thick. Geometrical parameters of the cutter cutting part  $\varphi = 45^\circ$ ;  $\alpha = 20^\circ$ ;  $\gamma = -10^\circ$ .

It is necessary to determine the feed  $f_c$  using a tabular method if it is known that the diameter of the part before processing is  $\varnothing 95$  mm. Processing in one pass up to a diameter  $\varnothing 87$  mm without liquid coolant emulsion should be done [62-64]. The length of processing is 85 mm, the overhang of the cutter is 30 mm, the allowable deflection of the cutter from the strength conditions of the holder  $f_c < 0.1$  mm. Determine if the assigned feed corresponds to the conditions of cutter rigidity, tip strength and, if necessary, correct it.

Determine the cutting speed corresponding to the maximum resource of the cutter, the period of its tool life; calculate the number of parts processed during the period of its tool life, as well as the need for a tool, if

it is necessary to process 1000 pieces of parts. Machine tool – model 16K20, cutter tool life – calculate.

5.4.4. On a lathe, semi-finishing ( $R_z$  20) processing of a shaft made of hard-to-cut steel DIN X12 CrNiTi 18-9(12X18H10T) is performed. Depth of cut  $d_c = 2$  mm.

1. Determine the brand of hard alloy material of the cutter and the chemical composition of the material. Choose liquid coolant emulsion for efficient material processing.

2. According to standard materials, select the optimal cutting conditions that provide a given tool life.

3. Calculate the cutting force  $P_z$  and the amount of heat released during processing.

5.4.5. During rough turning of shafts (diameter  $\varnothing 80$  mm; shaft material 37Cr4 (steel 40X GOST 4543-71); cutter working part material P10 ISO 513(T15K6)) on an automatic line, the cutter tool life  $T_1$  at cutting speed  $V_1$  dissatisfied with production due to two-hour line stops for tool changes. It is necessary to increase tool life by 1.5 times. How does this change the cutting speed? Determine  $V_1$  and  $V_2$ .

5.4.6. Determine the tangential component of the cutting force  $P_z$  when turning structural steel with a carbide straight-turning cutter with geometrical parameters and cutting modes:  
 $\varphi = 90^\circ$ ;  $\gamma = 0^\circ$ ;  $\lambda = 0^\circ$ ;  $d_c = 4$  mm;  $f_c = 2.5$  mm/rev;  $V = 160$  m/min;  
 $\sigma_u = 700$  MPa.

5.4.7. Determine the axial component of the cutting force  $P_a$  during turning of structural steel with a straight-turning cutter made of hard alloy with geometrical parameters:  
 $\varphi = 90^\circ$ ;  $\gamma = 0^\circ$ ;  $\lambda = 0^\circ$ ;  $d_c = 4$  mm;  $f_c = 2.5$  mm/rev;  $V = 160$  m/min;  
 $\sigma_u = 700$  MPa.

5.4.8. Calculate the value of the radial component of the cutting force  $P_r$  when turning structural steel with a carbide straight-turning cutter with

geometrical parameters and cutting modes:  
 $\varphi = 90^0$ ;  $\gamma = 0^0$ ;  $\lambda = 0^0$ ;  $d_c = 4$  mm;  $f_c = 2.5$  mm/rev;  $V = 160$  m/min;  
 $\sigma_u = 700$  MPa.

5.4.9. Determine the cutting power when turning corrosion-resistant steel with cutters from hard alloy K30-K40 ISO513 (VK8) with geometrical parameters and cutting modes:  
 $\varphi = 45^0$ ;  $\gamma = 10^0$ ;  $\lambda = 5^0$ ;  $d_c = 2$  mm;  $f_c = 0.7$  mm/rev;  $n = 400$  min<sup>-1</sup>;  
 $D = 200$  mm;  $\sigma_u = 540$  MPa.

5.4.10. Determine the cutting power  $N_c$  when turning gray cast iron HB190 with hard alloy cutters K20-K30 ISO513 (VK6) VK6: with geometrical parameters and cutting modes:  
 $\varphi = 60^0$ ;  $\gamma = -15^0$ ;  $\lambda = -5^0$ ;  $a_c = 1.2$  mm;  $b_c = 6.0$  mm;  $V = 90$  m/min;  
 $D = 200$  mm;  $\sigma_u = 320$  MPa.

5.4.11. Calculate the cross-sectional area of cutter tool holder with a tip of hard alloy P10 ISO 513(T15K6)T15K6, designed for rough turning new shaft  $\varnothing 100h12$  made from DIN C45 (steel 45) with tensile strength  $\sigma_u = 750$  MPa. Cutter geometry:  $\varphi = 45^0$ ;  $\varphi_1 = 15^0$ ;  $\gamma = 0^0$ ;  $\lambda = 0^0$ .

Workpiece diameter  $\varnothing 110$  mm; feed  $f_c = 0.8$  mm/rev; speed  $V = 120$  m/min; cutter overhang  $l_c = 50$  mm.

Cutter holder material – DIN C 50E (steel 50);  $\sigma_u = 650$  MPa,  $\sigma_b = 200$  MPa;

Permissible deflection of the cutter  $f_b = 0.1$  mm.

Calculate cross-sectional area for square and rectangular holders.

5.4.12. Calculate processing modes of sprocket type part (Fig. 18), manufactured on a CNC machine 16K20F3. Material part – 37Cr4 (steel 40X GOST 4543-71);

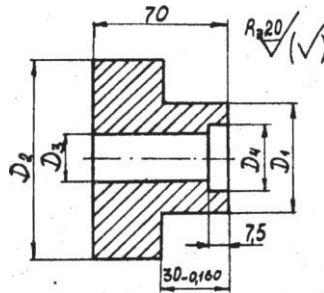


Fig. 18. Sketch of a workpiece for a machined sprocket part

Workpiece – forging (Fig. 19). Diametrical dimensions:  $D_1 = 70h10$ ;  $D_2 = 120$  mm;  $D_3 = 25h11^{(+0.13)}$ ;  $D_4 = 40h10^{(+0.1)}$ . Diametrical and end allowances:  $h_g = 5$  mm;  $h_T = 4$  mm;  $R_a = 6.3 \mu m$ .

5.4.13. Calculate the machining modes of the sprocket part type. (Fig. 18), made on CNC machine tool 16K20F3. Part material – DIN X19NiCrNo4 (steel 18X2H4MA);  $\sigma_u = 840$  MPa; workpiece – forging.

Diametrical and end allowances:  $h_g = 8$  mm;  $h_T = 4.5$  mm;  $R_a = 6.3 \mu m$ .

5.4.14. Determine the cutting conditions for circular internal grinding of a part of the "body of rotation" type. The material of the part is DIN 37Cr4 (steel 40X);  $\sigma_u = 620$  MPa; HRC 55. Diameter of the workpiece:  $D_w = 92$  mm; diameter of the machined hole.  $D_0 = 41.82$  mm; machining length  $l = 10$  mm. The diameter of the hole after machining is  $D_{0f} = 42.6^{+0.016}$ ; roughness  $R_a = 0.05 \mu m$ .

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## 6. OPTIMIZATION OF CUTTING MODES

### *6.1. A brief theoretical introduction*

One of the most common design tasks is to find the optimum operating conditions for the cutting system [64-68]. The search procedures includes:

- establishing the boundaries to be optimized technological system, within which experimental values can be found, or forming a set of technical constraints;
- determination of the quantitative criterion (optimization criterion, target function), on the basis of which you can make an analysis of the options in order to identify the best one;
- construction of mathematical model, which reflects interrelations between variables and represents set of equations and inequalities, reflecting target function and limitations.

Depending on the number of the target functions there are one- and multi-criteria optimization problems and depending on the type of the target function and constraints there are linear and non-linear optimization problems [69-71].

### *6.2. Typical task 8. Optimization turning modes using linear programming*

How to determine the optimum cutting speed and feed rate, providing maximum machining efficiency value (according to the machining time component).

Initial data; a machined detail – a shaft (Fig. 19); technological operation – rough turning; machine tool – screw-cutting lathe 16K20F3, workpiece – rolled, material DIN C 45 (steel 45);  $\sigma_u = 598$  MPa;

$D_1 = 200$  mm;  $\sigma_b = 200$  MPa; the tool – straight-turning lathe cutter, with mechanical fixing of hexagonal tip made of tungsten carbide P10 ISO 513 (T15K6):

$\varphi = 45^\circ$ ;  $\varphi_1 = 10^\circ$ ;  $\gamma = 10^\circ$ ;  $C_n = 5$  mm;  $B \times H = 25 \times 25$ ;  $l_c = 25$  mm;  
 $T = 60$  min;  $d_c = 3$  mm.

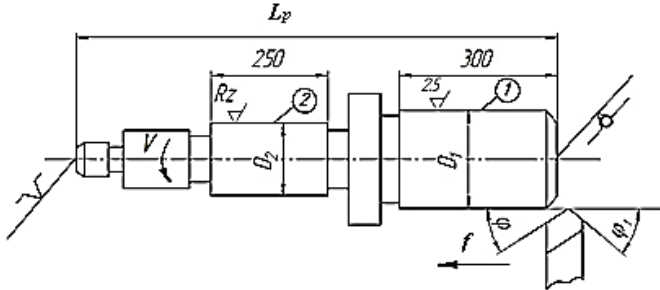


Fig. 19. Shaft processing scheme

### *Formation of a system of constraints*

1. Constraint on the cutting capabilities of the tool. To form this constraint, we use the known dependencies [45] and the information of a typical problem 6:

$$\left\{ \begin{array}{l} n \cdot f_c \leq \frac{318 \cdot C_v \cdot K_v}{T^m \cdot d_c^{x_v} D}; \\ n \cdot f_c^{0.45} \leq \frac{318 \cdot 340 \cdot 1.25}{60^{0.2} \cdot 3^{0.15} \cdot 200} = 252.69. \end{array} \right.$$

Here [45]:

$C_v = 340$ ;  $K_v = 0.15$ ;  $m = 0.2$ ;  $K_v = K_{mv} \cdot K_{nv} \cdot K_{lv} = 1.25 \cdot 1.0 \cdot 1.0 = 1.25$ ;

$$K_{mv} = K_r \cdot \left( \frac{750}{\sigma_u} \right)^{n_v} = 1.0 \cdot \left( \frac{750}{598} \right)^{1.0} = 1.25;$$

2. Power limitation of the main drive motor machine motion:

$$\left\{ \begin{array}{l} n^{n_{p_z}+1} \cdot f_c^{y_{p_z}} \leq \frac{1000^{n_p} \cdot 612 \cdot 10^4 \cdot N_e}{C_{p_z} \cdot K_{p_z} (\pi D)^{n_{p_z}+1} \cdot d_c^{x_{p_z}}}; \\ n^{0.85} \cdot f_c^{0.75} \leq \frac{1000^{0.85} \cdot 612 \cdot 10^4 \cdot 10 \cdot 0.8}{300 \cdot 0.84 \cdot (3.14 \cdot 200)^{0.85} \cdot 3} = 96.13. \end{array} \right.$$

Here [45]:  $C_{p_z} = 300$ ;  $x_{p_z} = 1.0$ ;  $y_{p_z} = 0.75$ ;  $n_{p_z} = -0.15$ ;  
 $K_{p_z} = K_{m_{p_z}} \cdot K_{\varphi_{p_z}} \cdot K_{\gamma_{p_z}} \cdot K_{\lambda_{p_z}} = 0.84$ ;  $N_e = N_t \cdot \eta$  – effective power of the drive, taking into account the efficiency  $\eta$ .

3. Constraint on the force allowed by the strength of the weak link of the machine feed mechanism [45]:

$$f_c^{y_{p_x}} \leq \frac{[P_f]}{9.81 \cdot C_{p_x} d_c^{x_{p_x}} \cdot K_{p_x}};$$

$$f_c^{0.5} \leq \frac{5884}{9.81 \cdot 339 \cdot 3^{1.0} \cdot 0.65} = 0.89.$$

Here  $[P_f]$  – the maximum allowable force for the feed ( $[P_f] = 5884$  N);

$C_{p_x} = 339$ ;  $x_{p_x} = 1.0$ ;  $y_{p_x} = 0.5$ ;  $K_{p_x} = K_{\varphi_{p_x}} \cdot K_{\gamma_{p_x}} \cdot K_{\lambda_{p_x}} = 0.65$ ;

4. Constraint on the strength of the cutter holder:

$$\left\{ \begin{array}{l} n^{n_{p_z}} \cdot f_c^{y_{p_z}} \leq \frac{1000^{n_{p_z}} \cdot \sigma_u \cdot W_c}{C_{p_z} \cdot K_{p_z} (\pi D)^{n_{p_z}} \cdot d_c^{x_{p_z}} \cdot l_c \cdot K_s}; \\ n^{-0.15} \cdot f_c^{0.75} \leq \frac{1000^{-0.15} \cdot 200 \cdot 3.3 \cdot 10^4}{10 \cdot 300 \cdot 0.84 \cdot 3^{1.0} \cdot (3.14 \cdot 200)^{-0.15} \cdot 25} = 32.57. \end{array} \right.$$

Here  $W_c = \frac{B \cdot H^2}{6}$  – section modulus of the holder,  $\text{mm}^3$ ;  $K_s$  – safety factor.

5. Constraint on the strength of the tip of the cutter cutting part (when using high-speed tool steel in the calculations do not take into account).

For given values of the tip thickness  $c_p = 5$  mm and the main angle in plan  $\varphi = 45^0$  this limitation is:

$$f_c^{y_{Px}} \leq \frac{34 \cdot c_p^{1.25} \left( \frac{\sin 60^0}{\sin \varphi} \right)^{0.8}}{C_{Pz} d_c^{(x_{Px}-0.77)} \cdot K_{Pz}};$$

$$f_c^{0.75} \leq \frac{34 \cdot 5^{1.25} \left( \frac{\sin 60^0}{\sin 45^0} \right)^{0.8}}{339 \cdot 3^{1.0-0.77} \cdot 0.84} = 0.92.$$

6. Constraint on the value of the minimum feed allowed by the kinematics of the machine 16K20F3:  $f_c \geq 0.05$  (passport data).

7. Constraint on the maximum feed:  $f_c \leq 2.8$ .

8. Constraint on the value of the minimum rotation frequency of the machine spindle:  $n \geq 12.5$ .

9. Constraint on the value of the maximum rotation frequency:  $n \leq 2000$ .

### *Setting the target function*

For a large number of production situations, as a target (objective) function  $F$ , it is advisable to choose the smallest machine processing time  $t_m$ , characterizing the performance of the process [72-74]:

$$F = t_m = \frac{L}{n \cdot f_c},$$

or when considering the processing length unit  $L$ :

$$F = t'_m = \frac{1}{n \cdot f_c}.$$

For more complex objective functions, using the example of the cost price for two straight-turning machining, the objective function takes the

form of a three-dimensional surface and its level lines and gradient lines [6].

### *Development of a mathematical model of the cutting process (roughing)*

For the case of turning roughing, the basis should be taken model  $A$ , which is refined in the process of forming the given above the constraints and takes the form:

$$\left. \begin{array}{l} x_1 + 0.45x_2 \leq 5.53 \\ 0.85x_1 + 0.75x_2 \leq 4.57 \\ 0.5x_2 \leq -0.12 \\ -0.15x_1 + 0.75x_2 \leq 3.48 \\ 0.75x_2 \leq -0.08 \\ x_2 \geq -3.0 \\ x_2 \leq 1.03 \\ x_1 \geq 2.53 \\ x_1 \leq 7.6 \\ F_0 = (x_1 + x_2) \rightarrow \max \end{array} \right\} A'$$

Here  $x_1 = \ln n$ ;  $x_2 = \ln f_c$ .

### *Graphical interpretation and determination of the optimal cutting conditions*

To find  $n_{opt}$  and  $f_{c\ opt}$  graphically, it is necessary to form a polygon of possible solutions to the system of constraints included in  $A'$ . In Fig. 20, straight lines inequalities of the system  $A'$  are shown in double logarithmic scales, and the  $ABCD$  region of possible solutions for that system corresponding to its inequalities is highlighted [75-77]. Boundary lines  $AB$ ,  $BC$ ,  $CD$  and  $DA$ ) intersecting with each other, form a polygon, each of the

points inside which satisfies the inequalities of all boundary lines of the system  $A'$  participating in its formation.

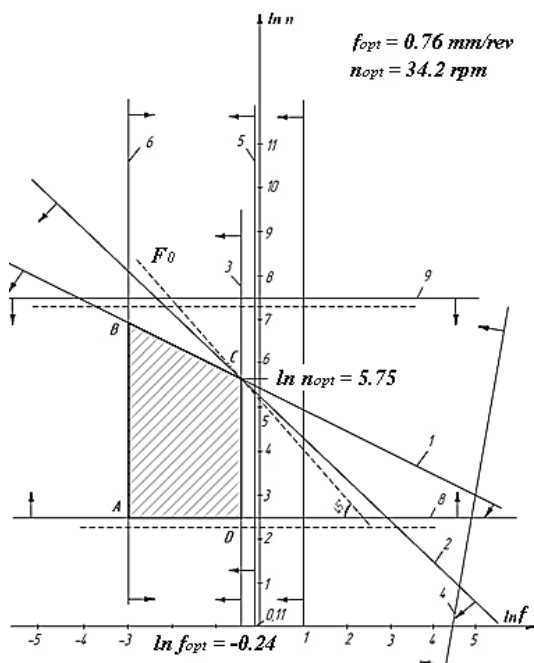


Fig. 20. Scheme for finding optimal cutting conditions for roughing

To find the optimal combination of elements  $\{n, f_c\}$ , it is necessary to determine at which of the points of the desired polygon  $ABCD$  the linear function of two variables  $F_0 = (x_1 + x_2)$  will take the maximum value. To do this, it is necessary to move the line  $F_0$  parallel to itself in the direction from the origin. In Fig. 20, at the vertex of polygon  $C$ , the objective function  $F_0$  takes on the largest value. Consequently, the vertex  $C$  is the optimum point, and its coordinate  $x_{1opt}$  and  $x_{2opt}$  is the optimal solution of the  $A'$  model.

Thus, the optimal combination of cutting mode elements for the case of roughing will be:

$f_c = 0.76 \text{ mm/rev}$ ;  $n = 314 \text{ min}^{-1}$  ( $V = 197.2 \text{ m/min}$ ). In accordance with the passport data of the machine, we have  $f_c = 0.76 \text{ mm/rev}$ ;  $n = 315 \text{ min}^{-1}$  ( $V = 198 \text{ m/min}$ ). The modes will correspond to the value of the main technological time for processing:

$$t_m = \frac{L}{n \cdot f_c} = \frac{300}{0.76 \cdot 315} = 1.24 \text{ min.}$$

The constructed polygon of possible solutions serves as the basis for choosing the most rational equipment for processing the part under consideration. For this case, it is necessary to select such a machine, the characteristics of which  $\{n_i, f_{ci}\}$  are closest to the polygon ABCD. Such is the machine 16K25:

$$n = 12.5 \dots 1600 \text{ min}^{-1}; f_c = 0.05 \dots 2.8 \text{ mm/rev}; N = 11 \text{ kW.}$$

Changing the model of the machine should be accompanied by a check on the effective power of the machine and the dimensions of the working area.

### ***6.3. Typical task 9. Optimization turning modes using geometric programming***

Determine the optimal modes of finishing turning of the shaft (Fig. 20) by the method of geometric programming (MGP) of zero degree of difficulty [78-81].

Initial data: processing area – 1; machine – lathe screw-cutting 16K20F3; workpiece – rolled steel DIN C 45 (steel 45);  $\sigma_u = 598 \text{ MPa}$ ;  $D = 100 \text{ mm}$ ;  $R_a = 2.5 \text{ } \mu\text{m}$  ( $R_z = 10 \text{ } \mu\text{m}$ );  $L_d = 850 \text{ mm}$ ;  $L = 250 \text{ mm}$ ; allowance  $\Delta = 1 \text{ mm}$ ; tool – straight-turning with mechanical fastening of a hexagonal tip made of P10 ISO 513 (T15K6) hard alloy;  $\varphi = 45^\circ$ ;

$\varphi_1=10^0$ ;  $\gamma=10^0$ ; transition radius of the wedge  $r=1.0$  mm;  $T$  – the numerical value of the tool life period (is not set, but is considered as some function  $T(V, f_c, d_c)$ ).

#### *Formation of the target function*

As an optimization criterion, we will take part of the cost ( $C$ ) of processing a part with a cutting tool in one pass, which depends on the speed of cut, feed and depth of cut. The analytical description of this dependence has the form (4):

$$C = A \cdot t_m + \frac{A \cdot t_{ch} \cdot t_m}{T}, \quad (4)$$

where  $A$  is the total cost of the machine and the machine operator, cent/min;  $t_{ch}$  – tool change time, min.

To solve this problem (taking into account the relation  $n=1000 \cdot V / (\pi \cdot D)$ ) compute  $t_m$  (5):

$$t_m = \frac{\pi \cdot D \cdot L \cdot \Delta}{1000 \cdot V \cdot f_c \cdot d_c}, \quad (5)$$

where  $\frac{\Delta}{d_c} = i$  – number of passes.

We represent the tool life equation for the turning operation in the following form (6):

$$T = \frac{C_T}{V^{\frac{1}{m}} \cdot f_c^{\frac{y_v}{m}} \cdot d_c^{\frac{x_v}{m}}}, \quad (6)$$

where  $C_T$  – coefficient that takes into account the working conditions of the tool  $y_v, x_v$ ;  $m$  – exponent, determined from the known tool life equations.

After substituting (5) and (b) into equation (4), we have

$$C = A \frac{\pi \cdot D \cdot L \cdot \Delta}{1000 \cdot V \cdot f_c \cdot d_c} + A \cdot t_{ch} \frac{\pi \cdot D \cdot L \cdot \Delta}{1000 \cdot C_T} \cdot V^{\frac{1}{m}-1} f_c^{\frac{y_v}{m}-1} d_c^{\frac{x_v}{m}-1}. \quad (7)$$

### *Formation of a system of technical constraints*

For finishing, the limitation on the precision characteristics of the part (tolerance field, the accuracy of the surfaces shape and their relative position is important [82-85]. In addition to the accuracy limit, we will also impose a limit on the maximum value of the depth of cut. These constraints should be presented explicitly.

1.  $P_y \leq [P_y]$ , where  $P_y$ ,  $[P_y]$  – radial component of cutting force and its maximum allowable value, respectively. For an explicit representation, it is necessary to use the relation:  $P_y = \frac{3 \cdot f_d \cdot E \cdot \pi \cdot D^4 \cdot \mu}{64 \cdot L_d^3}$

and analytical transformation, in the result of which we get:

$$V^{n_p} \cdot f_c^{y_p} \cdot d_c^{x_p} \leq \frac{3 \cdot \mu \cdot f_d \cdot E \cdot \pi \cdot D^4}{64 \cdot C_p \cdot K_p \cdot L_d^3}, \quad (8)$$

where  $n_p$ ,  $y_p$ ,  $x_p$  – exponents characterizing the influence on  $P_y$  the corresponding parameters: speed, feed and depth of cut value;

$\mu$  – coefficient taking into account the features of fixing the part. For the option fastenings in the cartridge and the center:  $\mu = 5.4$ .

Permissible arrow part deflection  $f_d$  (approximately 20% from tolerance and for  $R_z = 10 \mu\text{m} \rightarrow f_d = 0.05 \text{ mm}$ );

$C_p$  – coefficient taking into account the influence of working conditions on  $P_y$  force component, taken in the reference book as a basis;

$K_p$  – coefficient, taking into account the difference between specific working conditions from those given in the reference guide.

2. The maximum depth of cut  $d_c$  should not exceed the machining allowance, i.e.  $d_c \leq \Delta$ . For the above initial data, the components of the dependence (8) according to [45] take the values:  $C_{Py} = 243$ ;  $K_{Py} = 1.0$ ;  $x_{Py} = 0.9$ ;  $y_{Py} = 0.6$ ;  $n_{Py} = -0.3$ . Then the dependence (8) is represented as  $V^{-0.3} \cdot f_c^{0.6} \cdot d_c^{0.9} \leq 0.17$ , and the constraint on depth of cut  $d_c \leq 1$ .

### *Formation of the mathematical model of the problem*

It is necessary to use the formulation given in [86, 87], and when forming the dual problem, take into account the features of the objective function (two-term posinom) using the toolkit of method of geometric programming (GP).

Direct setting of MGP:

Minimize:

$$g_0(V, f_c, d_c) = C = 46.63 \cdot V^{-1} \cdot f_c^{-1} \cdot d_c^{-1} + 0.82 \cdot 10^{-12} V^4 \cdot f_c^{1.25} \cdot d_c^{0.75},$$

under the constraints:  $577 \cdot V^{0.3} \cdot f_c^{0.6} \cdot d_c^{0.9} \leq 1$ ;  $d_c \leq 1$ .

It should be noted that this direct problem is a GP-problem of zero degree of difficulty, and each constraint contains a single posinomial term:

$$\lambda_k = \Delta_k = w_k \text{ (for } k=1, 2)$$

The dual GP problem with constraints:

Maximize:

$$V(w) = \left( \frac{C_{01}}{w_{01}} \right)^{w_{01}} \cdot \left( \frac{C_{02}}{w_{02}} \right)^{w_{02}} \cdot C_{11}^{w_{11}} \cdot C_{21}^{w_{21}}, \quad (9)$$

under the constraints:

$$\begin{aligned}
V: & -w_{01} + 4w_{02} - 0.3w_{11}; \\
f_c: & -w_{01} + 1.25w_{02} - 0.6w_{11}; \\
d_c: & -w_{01} + 0.75w_{02} - 0.9w_{11} + w_{21}.
\end{aligned} \tag{10}$$

It should be noted that the sequence of solving the GP-problem with numerous posinomials is similar to the same GP problem with one-term posinomials.

1. Solve the system of linear equations (10). In this setting, the system has a unique solution:  $w_{01} = 0.76$ ;  $w_{02} = 0.24$ ;  $w_{11} = 0.67$ ;  $w = 0.21$ .

The feature of this option is the possibility already at the first stage of the decision, evaluate the contribution of each component of the objective function to the total cost  $C$  [41]. The cost of the first component associated with machining is 76%, and the component associated with tool change is 24%.

2. Calculate the extremum of the objective function (7). To do this, we find the maximum of the dual function (10):

$$\max V_g(w) = \left( \frac{46.63}{0.76} \right)^{0.76} \cdot \left( \frac{0.82 \cdot 10^{-12}}{0.24} \right)^{0.24} \cdot 5.77^{0.67} \cdot 1.0^{0.21} = 0.13.$$

3. Determine the optimal values of the cutting mode elements. Using the GP-relations [78], we compose a system of equations for determining the optimal cutting modes:

$$\begin{aligned}
0.13 \cdot 0.76 &= 46.63 \cdot V^{-1} \cdot f_c^{-1} \cdot d_c^{-1}; \\
0.13 \cdot 0.24 &= 0.82 \cdot 10^{-12} \cdot V^4 \cdot f_c^{1.25} \cdot d_c^{0.75}; \\
1.0 &= 5.77 \cdot V^{-0.3} \cdot f_c^{0.6} \cdot d_c^{0.9}; \\
1.0 &= d_c.
\end{aligned}$$

As a result of solving this system of equations, the optimal values of the elements of cutting modes are obtained:

$$V_{\text{opt}} = 421.39 \text{ m/min}; f_{c \text{ opt}} = 1.12 \text{ m/rev}; d_{c \text{ opt}} = 1.0 \text{ mm}.$$

#### 6.4. Typical task 10. Optimization turning modes using method of Lagrange multipliers

Determine the optimal cutting modes by the method of Lagrange multipliers [88-90].

Initial data: operation – finishing turning of a smooth shaft ( $d = 100$  mm,  $l = 600$  mm) on a 16K20 machine tool with a mechanically fastened hexagonal tip made of P10 ISO 513 (T15K6) hard alloy. The parameters of the cutter are determined by the cost of the tool reduced to one period of tool life  $a' = 10$  cent; cutting wedge rounding radius  $r = 1.0$  mm; geometric parameters of the cutter:

$\varphi = 45^\circ$ ;  $\varphi_1 = 10^\circ$ ;  $\gamma = 10^\circ$ ;  $B \times H = 25 \times 25$  mm.  $l_c = 50$  mm;  $c_n = 5$  mm.

Turning produced in centers with a depth of cut  $d_c = 0.5$  mm; roughness  $R_a = 2.5$   $\mu\text{m}$  and an accuracy of diametrical size according to the 10th grade (size error value 0.12 mm).

The kinematics of the machine allows the carriage feed within  $0.05 \leq f_c \leq 2.5$  mm/rev and spindle speed within  $12.5 \leq n \leq 1600$   $\text{min}^{-1}$ .

Allowable for the strength of the feed mechanism cutting force  $P_a = 6000$  N; main drive power  $N = 10$  kW; drive efficiency factor  $\eta = 0.8$ ; compliance of the lathe system in the weakest section  $\omega_{\max} = 0.096$   $\mu\text{m/N}$ ; [5], cost price per machine minute  $a = 1.0$  cent; , tool change time  $t_{ch} = 2.0$  min.

Tool life period:

$$T = \frac{C_T \cdot K_T}{V^m \cdot f_c^y \cdot d_c^x},$$

where  $m = 5$ ;  $n = 1.0$ ;  $x = 0.75$ ;  $C_T = 420^5 = 1.307 \cdot 10^{13}$ ;  $K_T = 1.0$ .

The cutting force is determined from the expression:

$$P_z = C_{Pz} \cdot K_{Pz} \cdot V^{\alpha_1} \cdot f_c^{\alpha_2} \cdot d_c^{\alpha_3},$$

where  $C_{P_z} = 300$ ;  $K_{P_z} = 1.0$ ;  $\alpha_1 = -0.15$ ;  $\alpha_2 = 0.75$ ;  $\alpha_3 = 1.0$ .

Let us formulate the range of permissible values of cutting modes. To do this, we define a set of constraints imposed on the cutting modes.

1. Constraint on the power  $N$  ( $N = 10$  kW) of the drive electric motor for the main movement of the machine. This constraint is formed taking into account the power spent on the cutting process  $N_c$ . Let's take the drive efficiency  $\eta = 0.8$ .

$$N_c = \frac{P_z \cdot V}{60 \cdot 1020} = \frac{V^{0.85} \cdot f_c^{0.75} \cdot d_c^{1.0}}{20.4} \leq N \cdot \eta = 8.0; \quad (11)$$

$$V^{0.85} \cdot f_c^{0.75} \cdot d_c^{1.0} \leq 163.2.$$

2. Constraint on the force  $P_x$  ( $P_x = 0.25 \cdot P_z$ ) permissible by the strength of the weak link of the machine feed mechanism:

$$P_x \leq P_{pm} \rightarrow 0.25 \cdot P_z \leq P_{pm} \rightarrow P_z \leq 2400 \text{ N.}$$

3. Tool holder strength constraint:

$$M_b = P_z \cdot l_c \leq \frac{\sigma_b \cdot W_c}{K_s},$$

where  $M_b$  – the bending moment from the force  $P_z$ ;  $l_c$  – the cutter overhang;  $\sigma_b$  – ultimate tensile strength of the cutter when bending;

$W_c$  – section modulus of the cutter holder:  $W_c = \frac{B \cdot H^2}{6}$ ;  $K_s$  – safety factor

( $K_s = 1.5 \dots 2.0$ ).

Let's take  $\sigma_b = 600$  MPa;  $K_s = 1.5$ . Then

$$300 \cdot V^{-1.5} \cdot f_c^{0.75} \cdot d_c^{1.0} \leq \frac{600 \cdot 25 \cdot 25^2}{1.5 \cdot 6} \rightarrow P_z \leq 20830 \text{ N.}$$

4. Constraint on the strength of the carbide-cutting insert

$$P_z \leq 34 \cdot d_c^{-0.77} \cdot c_n^{1.25} \cdot \left( \frac{\sin 60^\circ}{\sin \varphi} \right)^{0.8} = 340 \cdot 0.5^{-0.77} \cdot 5^{1.25} \cdot 1.5^{0.8} = 6000 \text{ N,}$$

where  $c_n$  – thickness of the carbide tip.

5. Constraint on the permissible rigidity of the cutter

$$P_z \leq \frac{3 \cdot f_t \cdot E \cdot J}{l_c^3} = \frac{3 \cdot 0.05 \cdot 2.1 \cdot 10^5 \cdot 25 \cdot 25^3}{50^3 \cdot 12} = 13020 \text{ N},$$

where  $f_t$  – permissible deflection of the cutter, mm;  $J = \frac{bh^2}{12}$  – the polar inertia moment of the tool holder section, mm<sup>3</sup>;  $E$  – modulus of elasticity of the tool holder, MPa.

6. The constraint on the rigidity of the detail. This constraint takes into account the maximum allowable deflection  $f_d$  of the detail, which is set depending on the tolerance  $\Delta_j$  on the size of the detail:  $f_d = (0.25 \dots 0.5) \cdot \Delta_j$ .

Deflection of a part under the action of a concentrated force  $R$ :

$R = \sqrt{P_z^2 + P_y^2} \approx 1.1 \cdot P_z$  is equal to:

$$f_d = \frac{R \cdot L_d^3 \cdot \mu}{K_{fd} \cdot E \cdot J_d},$$

where  $L_d$  – distance from the support to the section under consideration, for this calculation option. We take  $L_d = 400$  mm;  $J_d$  – a polar moment of inertia, mm<sup>4</sup>;  $\mu$  – dynamic coefficient;  $K_{fd}$  – coefficient depending on the method of fastening the detail ( $K_{fd} = 2.4 \dots 3$  – when fastening the detail cantilever in the chuck,  $K_{fd} = 70 \dots 100$  when fastening in the centers,  $K_{fd} = 130 \dots 140$  when fixing in the chuck with preloading the center rear tailstock). In our case, the constraint has the form:

$$P_z \leq \frac{f_d \cdot K_{fd} \cdot E \cdot J_d}{1.1 \cdot L_d^3 \cdot \mu} = \frac{0.03 \cdot 100 \cdot 2.1 \cdot 10^5 \cdot 0.05 \cdot 100^4}{1.1 \cdot 400^3 \cdot 1.2} = 3550 \text{ N}.$$

7. The constraint on the roughness of the machined surface. We use the empirical equation [45]:

$$R_a = 0.16 \cdot f_c^{0.59} \cdot V^{-0.19} \cdot r^{-0.29} \cdot (90 + \gamma)^{0.86} \leq R_{a \text{ pm}},$$

and theoretical dependence:

$$R_z = r - \sqrt{r^2 - \left(\frac{f_c}{2}\right)^2}.$$

The actual values of  $R_z$  are approximately 1.4 times more than theoretically calculated. Taking  $R_z = 5R_a$ , we get:

$$f_{c\max} = 2\sqrt{(1 - R_z)^2 - 1} \approx 0.36 \text{ mm/rev}.$$

Thus, a mathematical model for calculating the optimal cutting modes will be:

$$C = \frac{1 + \frac{12+10}{1.07 \cdot 10^{13}} \cdot V^5 f_c^{0.75} d_c^{1.0}}{V \cdot f_c} \rightarrow \min;$$

$$P_z = 300 \cdot V^{-0.15} f_c^{0.75} d_c^{1.0} \leq 3550 \rightarrow V^{-0.15} \cdot f_c^{0.75} \cdot d_c^{1.0} \leq 11.8;$$

$$\frac{P_z \cdot V}{1020 \cdot 60} \leq 8 \text{ kW} \rightarrow V^{0.85} \cdot f_c^{0.75} \cdot d_c^{1.0} \leq 1632;$$

$$R_a = 3.55 \cdot V^{-0.19} \cdot f_c^{0.59} \leq 2.5 \rightarrow V^{-0.19} \cdot f_c^{0.59} \leq 0.75;$$

$$f_c \leq 2.8; f_c \geq 0.05; V \leq 502.4; V \geq 3.9.$$

We calculate the optimal cutting mode with active force constraint taking into account the value of the permissible tangential component  $P_{zpm}$ :

$$P_{zpm} \geq P_z = K_{Pz} \cdot C_{Pz} \cdot V^{\alpha_1} \cdot f_c^{\alpha_2} \cdot d_c^{\alpha_3}.$$

In [68] the optimal values of cutting speed  $V_0$  and feed  $f_{c0}$  are determined with this constraint:

$$\begin{aligned}
V_0 &= \left( P_{z0}^{-n} \cdot l^{mx+\alpha_1 n} \cdot d_c^{-kx-ny} \right)^{\frac{1}{mx}}; \\
f_{c0} &= \left( P_{z0} \cdot l^{-\alpha_1} \cdot d_c^y \right)^{\frac{1}{x}}; \\
P_{z0} &= \frac{P_{zpm}}{K_{Pz} \cdot C_{Pz}}; \\
l &= \left( \frac{\alpha \cdot (\alpha_2 - \alpha_1)}{b \cdot (\alpha_2 \cdot (m-1) - \alpha_1 \cdot (n-1))} \right)^{\frac{1}{m}}; \\
x &= \frac{\alpha_2 \cdot m - \alpha_1 \cdot n}{m}; \\
y &= \frac{\alpha_1 \cdot k - \alpha_3 \cdot n}{m}.
\end{aligned}$$

Components of the above expressions for this case take the following values:

$$\begin{aligned}
x &= \frac{0.75 \cdot 5 - (-0.15) \cdot 1}{5} = 0.78; \quad y = \frac{(-0.15) \cdot 0.75 - 1 \cdot 5}{5} = -1.02; \\
\ell &= \left( \frac{1 \cdot (0.75 - (-0.15))}{9.18 \cdot 10^{13} (0.75 \cdot (5-1)) - (-0.15 \cdot (1-1))} \right)^{\frac{1}{5}} = 200.84; \\
f_{c0} &= \left( 1.18 \cdot 200.84^{0.15} \cdot 0.5^{-1.02} \right)^{\frac{1}{0.78}} = 8.34 \text{ mm/rev}; \\
V_0 &= \left( 1.18^{-1} \cdot 200.84^{3.75} \cdot 0.5^{0.435} \right)^{\frac{1}{0.78 \cdot 5}} = 145.18 \text{ m/min}.
\end{aligned}$$

With an active power (capacity) constraint, taking into account the maximum permissible (overload) capacity  $N_{pm}$ , we have:  $N_{pm} = \frac{P_z \cdot V}{60 \cdot 1020}$ . The optimum cutting modes are determined from expressions similar to

(11) in which parameter  $P_{z0}$  is replaced by  $N_{0_{pm}} = \frac{N_{pm} \cdot 60 \cdot 1020}{K_{Pz} \cdot C_{Pz}}$  and parameter  $\alpha_1$  is replaced by  $\alpha_1 + 1$ .

At a given variant of the initial data components  $x$ ,  $y$  and  $l$  take the following values

$$x = \frac{0.75 \cdot 5 - 0.85 \cdot 1}{5} = 0.58; \quad y = \frac{0.85 \cdot 0.75 - 1 \cdot 5}{5} = -0.87;$$

$$\ell = \left( \frac{1 \cdot (0.75 - 0.85)}{9.18 \cdot 10^{13} (0.75 \cdot (5 - 1)) - (0.85 \cdot (1 - 1))} \right)^{\frac{1}{5}} = -3.63^{0.2} \cdot 10^2 < 0.$$

The last inequality means that the points of tangency of the level lines of the objective function and line of constraint on the permissible drive power of the main motion do not exist [68, 91, 92].

Consider the surface roughness constraint:

$$x = \frac{0.59 \cdot 5 - (-0.19)}{5} = 0.628; \quad y = \frac{(-1.19) \cdot 0.75 - 0.5}{5} = -0.03;$$

$$\ell = \left( \frac{1 \cdot (0.59 - (-0.19))}{9.18 \cdot 10^{13} (0.59 \cdot (5 - 1)) - (-0.19 \cdot (1 - 1))} \right)^{\frac{1}{5}} = 204.77;$$

$$f_{c0} = \left( 0.75 \cdot 204.77^{0.19} \cdot 0.5^{-0.03} \right)^{\frac{1}{0.628}} = 3.16 \text{ mm/rev};$$

$$V_0 = \left( 0.75^{-1} \cdot 204.77^{2.95} \cdot 0.5^{-0.47} \right)^{0.32} = 185.97 \text{ m/min}.$$

We calculate the optimal cutting speed for  $f_c = f_{cmax} = 0.36 \text{ mm/rev}$ . Here  $f_{cmax}$  is the maximum allowable feed when turning with a roughness  $R_a = 2.5 \text{ } \mu\text{m}$  [45]:

$$V_0 = \left( \frac{1}{9.18 \cdot 10^{13} \cdot 4} \cdot 0.36^{-1} \cdot 0.5^{-0.75} \right) = 263.58 \text{ m/min}.$$

We calculate the coordinates of the intersection point for the constraint lines in  $f_{cmax}$  and  $N_{pm}$ . To do this, solve the system of equations:

$$\begin{cases} f_c = f_{cmax} = 0.36; \\ V^{0.85} f_c^{0.75} d_c^{1.0} = 163.2 \end{cases} \rightarrow V = \left( \frac{326.4}{0.36^{0.75}} \right)^{\frac{1}{0.85}} = 223.2 \text{ m/min.}$$

We define the coordinates of the intersection point of the restriction lines in  $P_{zpm}$  and  $N_{pm}$ :

$$\begin{cases} V^{-0.15} f_c^{0.75} d_c^{1.0} = 0.69; \\ V^{0.85} f_c^{0.75} d_c^{1.0} = 163.2 \end{cases} \rightarrow \begin{cases} f_c = \left( 1.38 V^{0.15} \right)^{\frac{1}{0.75}} = 4.57 \text{ mm/rev}; \\ V = \frac{326.4}{1.38} = 236.5 \text{ m/min.} \end{cases}$$

Let the calculations show that the best solution that satisfies all the constraints is a pair:  $\{f_c = 0.36 \text{ mm/rev}; V = 264 \text{ m/min}\}$ . We will correct the cutting modes. The closest lower feed value allowed by the machine kinematics is  $f_c = 0.35 \text{ mm/rev}$ . Then the optimal value of the cutting speed:  $V = 265 \text{ m/min}$  ( $n = 844 \text{ min}^{-1}$ ). The closest rotation speed:  $n_1 = 800 \text{ min}^{-1}$  ( $V_1 = 251 \text{ m/min}$ ),  $n_2 = 1000 \text{ min}^{-1}$  ( $V_2 = 314 \text{ m/min}$ ).

As an optimization criterion (target function) we will use the technological cost of performing transitions  $C$  (the calculation uses the initial data presented in the task 6.2.3). After a number of transformations, this criterion has the form:

$$C = \frac{a + b \cdot V^m \cdot f_c^y \cdot d_c^x}{V \cdot f_c}; \quad b = \frac{a \cdot t_{ch} + a'}{K_T \cdot C_T}; \quad T = \frac{K_T \cdot C_T}{V^m f_c^y \cdot d_c^x},$$

where  $a$  – cost price machine-minute, cent;  $a'$  – the cost of the tool reduced to a single tool life period;  $t_{ch}$  – tool change time;  $T$  – tool cost price;  $K_T$ ,  $C_T$  – normative factors;  $V$  – cutting speed;  $f_c$  – tool feed;  $d_c$  – depth of cut.

We calculate the cost price of the technological pass at these two points  $n_1 = 251 \text{ m/min}$ ;  $n_2 = 314 \text{ m/min}$ :

$$C_1 = \frac{1 + 9.18 \cdot 10^{-13} \cdot 0.35^1 \cdot 0.5^{0.75} \cdot 251^5}{0.35 \cdot 251} = 21.67;$$

$$C_2 = \frac{1 + 9.18 \cdot 10^{-13} \cdot 0.35^1 \cdot 0.5^{0.75} \cdot 314^5}{0.35 \cdot 314} = 53.05.$$

The calculation result shows that the optimal values of the cutting modes:  $\{f_{c0} = 0.35 \text{ mm/rev}, n_0 = 800 \text{ min}^{-1}\}$ .

If the cutting speed changed continuously, as, on modern CNC machines, the correction of the modes should be carried out according to the number of parts processed for one period of tool life. We perform the necessary transformations:

$$\begin{aligned} T &= k \cdot t_m \rightarrow \frac{C_T \cdot K_T}{V^m \cdot f_c^y \cdot d_c^x} = \frac{k \cdot (\pi \cdot D \cdot l_d)}{1000 \cdot V \cdot f_c} \rightarrow V^{m-1} = \frac{C_T \cdot K_T \cdot 1000}{k \cdot (\pi \cdot D \cdot l_d \cdot f_c^{y-1} d_c^x)} = \\ &= \frac{1.307 \cdot 10^{12}}{k \cdot (3.14 \cdot 8 \cdot 0.59)} = \frac{8.8 \cdot 10^{10}}{k}. \end{aligned}$$

We select such values  $k$  that approximate  $V$  the smallest value above and below. At  $k = 18$ , the cutting speed  $V = 254.4 \text{ m/min}$ ; at  $k = 17$ , the cutting speed  $V = 268.3 \text{ m/min}$ . Lower cost price corresponds to the first value  $V = 254.4 \text{ m/min}$ . Then the corrected cutting mode corresponds to  $\{f = 0.35 \text{ mm/rev}; V = 254.4 \text{ m/min}\}$ .

### **6.5. Typical task 11. Multi-criteria optimization turning modes using climb-down method**

Determine the best cutting conditions using multi-criteria optimization (climb-down method) [68, 93].

Initial data: detail to be machined – shaft (Fig. 19); operation – finishing turning;  $R_a = 2.5 \text{ }\mu\text{m}$ ; machine – screw-cutting lathe 16K20F3; workpiece – rolled metal, DIN C 45;  $\sigma_u = 598 \text{ MPa}$ . Detail geometric parameters:  $D_1 = 100 \text{ mm}$ ;  $l_p = 860 \text{ mm}$ ,  $L = 250 \text{ mm}$ , allowance  $\Delta = 1.0 \text{ mm}$ ; tool – straight-turning cutter with mechanical fastening of a hexagonal tip made of hard alloy P10 ISO 513 (T15K6); geometric parameters of the cutter:  $\varphi = 45^\circ$ ;  $\varphi_1 = 10^\circ$ ; wedge transition radius  $r = 1.0$

mm;  $\gamma = 10^\circ$ ; tool life period  $T = 60$  min. Processing is carried out in one pass with depth of cut  $d_c = \Delta = 1$  mm.

Optimization criteria – machine time  $t_m$  and minimum deflection of the part  $f_d$ .

In accordance with the algorithm of the method of successive concessions, it is necessary to implement the following steps [68, 93].

1. Rank private performance criteria. Let us introduce the priority of the performance criterion (expressed in terms of machine time  $t_m$ :  $= F_1$ ).

2. Find the minimum value of the machine processing time of the shaft with constraints on the cutting capabilities of the tool  $V \leq V_T$  and on the allowable value of the surface roughness  $R_z \leq R_{zpm}$ . When setting the problem and forming the constraints, we will use the data [45], the method of forming constraints [75], and the method of geometric programming (GP) of zero degree of difficulty [68]. For this case of processing, the mathematical model of the GP problem in the direct formulation is given below.

Direct statement:

$$\text{minimize } F_1 = t_m = 250 \cdot f_c^{-1} \cdot n^{-1},$$

in the case of constraints:

$$0.0017 \cdot f_c^{0.2} \cdot n \leq 1;$$

$$2.04 \cdot f_c^{0.7} \cdot n \leq 1;$$

It should be noted that in this problem there are three posinomial terms and two variables ( $f, n$ ), which characterizes it as a task zero degree of difficulty (the number of unique members is one more number of variables). Each constraint contains a single posinomial member. The above direct statement of the GP corresponds to dual GP problem with constraints [78].

Dual statement:

maximize  $V(w) = 250^{w_1} \cdot 0.0017^{w_{11}} \cdot 2.04^{w_{21}} \cdot 0.0017^{w_{11}} \cdot 2.04^{w_{21}}$ , in the case of normalization constraints:  $w_{01} = 1$ , and orthogonality:

$$\begin{cases} f_c : & -w_{01} + 0.2 \cdot w_{11} + 0.7 \cdot w_{21} = 0; \\ n : & -w_{01} + w_{11} = 0. \end{cases}$$

Using the corresponding GP program of zero degree of difficulty, we determine:

- values of dual variables:  $\{w_{01} = 1; w_{11} = 1; w_{21} = 1.14\}$ ,
- maximum of the dual function:

$$\max V(w) = 250 \cdot 0.0017 \cdot 2.04^{1.14} = 0.96.$$

Optimal values of the elements of cutting modes from the conditions of invariance:

$$\begin{cases} 0.96 = 250 \cdot f_c^{-1} \cdot n^{-1}; \\ 1/1 = 0.0017 \cdot f_c^{0.2} \cdot n; \\ 1.14/1.14 = 2.04 \cdot f_c^{0.7} \cdot n^0. \end{cases}$$

The only solution to this system of equations is  $f_{c10} = 0.36$  mm/rev;  
 $n_{10} = 717.37$  rpm;  $V_{10} = 225.25$  m/min.

It must be remembered that the maximum of the dual function according to the GP theory corresponds to the minimum of the original objective function, i.e.  $t_m = 0.96$  min.

3. Solve the optimization problem according to the second efficiency criterion  $F_2$ , i.e. find such cutting conditions  $\{f_c; n\}$  which would minimize the deflection of the part  $f_d = F_2$  under the same constraints on the cutting capabilities of the tool and on the permissible roughness (stage 2). The problem statement in accordance with the GP methodology and data [68, 94] is presented below:

$$\begin{aligned}
F_2 = f_d &= 0.01 \cdot n^{-0.3} f_c^{0.6} \rightarrow \min; \\
0.017 \cdot f_c^{0.2} \cdot n &\leq 1; \\
2.04 \cdot f_c^{0.7} \cdot n^0 &\leq 1.
\end{aligned} \tag{14}$$

The GP method is inapplicable in such a formulation, since there are sign changes in the line of dual variables for  $f_c$ , which leads to the appearance of a negative dual weight  $w_{21}$ . This contradicts the original constraint of the GP method:  $w \geq 0$ . An analysis of various cutting schemes showed that this situation is typical for various combinations of initial data and various machining schemes (turning with transverse feed, threading, etc.), which are reduced to the statement [41, 95, 96].

To solve the problem of finding the minimum deflection  $f_d$ , we use the linear programming method [52]. The mathematical model of this method (after reduction to a linear form) is a set of linear inequalities and the objective function  $F_2$  obtained as a result of the logarithm  $e$  for the initial constraint expressions (14) and constraints on the kinematic capabilities of the machine 16K20F3:  $\{f_{c\min} \leq f_c \leq f_{c\max}; n_{\min} \leq n \leq n_{\max}\}$ :

$$\begin{cases}
x_1 + 0.2 \cdot x_2 \leq 6.38; \\
0.7 \cdot x_2 \leq 0.71; \\
x_1 \leq 7.6; \\
x_1 \geq 2.53; \\
x_2 \leq 1.03; \\
x_2 \geq -3.0; \\
-4.61 - 0.3 \cdot x_1 + 0.6 \cdot x_2 \rightarrow \min.
\end{cases} \tag{15}$$

Here the designations  $\{\ln n = x_1; \ln f_c = x_2\}$  are introduced. For implementation graphical method, as the most illustrative, the given inequalities and equations should be drawing as straight lines in the coordinate system " $n - f_c$ " (Fig. 22). On the graph, it is necessary to plot straight lines in double logarithmic scales and indicate with arrows on

which side of each straight line there are points corresponding to the allowable values of  $n$  and  $f_c$ .

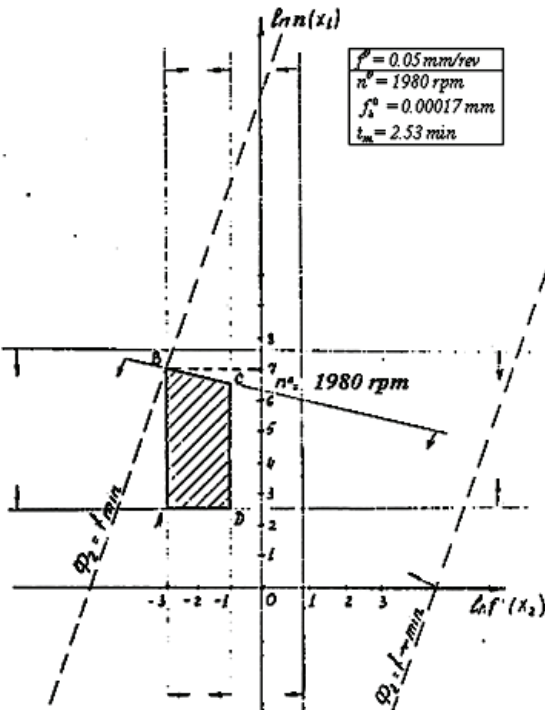


Fig. 21. Scheme for finding the optimal cutting conditions by the criterion  $f_d$

In this problem, the system of constraints is consistent, the admissible region  $ABCD$  (Fig. 21) is a closed polyhedron, and the dependence (15) to be minimized is shown by a dashed line  $f_d$ . Will the linear function be at its minimum when it passes through point  $B$  polygon of possible solutions. The coordinates of this point will give optimal solutions:  $f_{c0} = 0.05 \text{ mm/rev}$ ;  $n = 1980 \text{ rpm}$ ; bending deflection  $f_d = 0.00017 \text{ mm}$ ; machine time value  $t_m = 2.53 \text{ min}$ .

The analysis of the obtained results shows that the desire to minimize the machine time of processing and the deflection of the part leads to two sets of optimized parameters and two different values of the machine time:

$$\{f_{c10} = 0.36; n_{10} = 717.37\} \rightarrow t_{m1} = 0.96 \text{ min};$$

$$\{f_{c20} = 0.05; n_{20} = 1980\} \rightarrow t_{m2} = 25.3 \text{ min}.$$

To find a compromise variant of the implementation of processing modes, it is necessary to perform the fourth stage of solving the problem by the climb-down method [93, 97, 98].

4. Choosing a climb-down according to the first criterion of efficiency  $t_m$ . When choosing a climb-down, you should compare the value of machine time obtained as a result of single-criterion optimization (step 2) with the “tabular” value of machine time. For this processing case, by the data [68], we obtain tabular values:

$$f_c^T = 0.25 \text{ mm/rev}; V^T = 268 \text{ m/min}; t_m^T = 1.17 \text{ min}.$$

Then the climb-down  $h$  according to the first criterion is defined as the difference between the table and optimal (according to the first criterion) machine time:

$$h_1 = t_m^T - t_{m1} = 1.17 - 0.96 = 0.21 \text{ min}.$$

5. Find the minimum value of the detail-bending arrow  $f_d = F_2$ . With a new system of constraints, including the initial constraints on the cutting capabilities and the allowable roughness of the machined surface, as well as an additional constraint imposed on the value of the first criterion:  $F_1 \leq F_1^{**}$ , where  $F_1^{**} = F_1 + h_1 = 0.96 + 0.21 = 1.17 \text{ min}$ . This criterion transfer technique in constraint in another context has been considered [99-101]. You should compare two options for the same technique and identify the difference in their use.

For this option, the search for an extremum  $f_d$  is possible using the GP method of the first degree of difficulty [68]. Consider the mathematical model of the problem at this stage.

Direct statement of the problem:

Minimize:  $F_2 = f_d = 0.01 \cdot n^{-0.3} f_c^{0.6} \rightarrow \min$ ;  
under constraints:

$$\begin{aligned} 0.0017 \cdot f_c^{0.2} \cdot n &\leq 1; \\ 2.04 \cdot f_c^{0.7} \cdot n^0 &\leq 1; \\ 213.68 \cdot f_c^{-1} \cdot n^{-1} &\leq 1. \end{aligned}$$

Dual statement of the problem:

Maximize:  $V(w) = 0.01^{w_{01}} \cdot 0.0017^{w_{11}} \cdot 2.04^{w_{21}} \cdot 213.68^{w_{31}}$ ;  
under constraints:  $w_{01} = 1$ ;

$$\begin{aligned} -0.3w_{01} + w_{11} - w_{31} &= 0; \\ 0.6w_{01} + 0.2w_{11} + 0.7w_{21} - w_{31} &= 0. \end{aligned} \quad (16)$$

To solve the GP problem of the first degree of difficulty, it is recommended to use a machine-oriented dual method [68]. To do this, we choose the basic dual weights  $w_i = \{w_{01}, w_{11}, w_{21}\}$  and solve the system of equations (16) to the basis variables [the number of basis weights is one more than the number of optimized parameters and is equal to the number of system equations (16)]. In the case under consideration, linear dependences on  $w_{31}$ :

$$w_{11} = w_{31} + 0.3; \quad w_{21} = 1.14w_{31} - 0.94; \quad w_{01} = 1.0.$$

Dual weight  $w_{01}$  determine according to the condition of normalization.

For everyone  $w_i$  to be positive, there  $w_{31}$  must be more than  $w_{31} = 0.82$ . As a first approximation, we take  $w_{31} = 0.82$ . We calculate the maximum value of the dual function:  $V(w_{31})$ :

$$V(w_{31}) = 0.01 \cdot 0.0017^{1.14} \cdot 2.04^0 \cdot 213.68^{0.82}.$$

The maximum of the dual function is equal to the minimum of the “direct”, i.e.  $f_d = 0.000656$  mm, which is significantly lower than the permissible detail climb-down ( $f_d = 20\%$  tolerance fields and at  $R_z = 10 \rightarrow f_{d\text{ pm}} = 0.02$  mm).

To search  $\max V(w_{31})$ , you can use the program of numerical optimization (dichotomy, golden ratio, Fibonacci). The appeal to the standard dichotomy program is described in [41]. To implement the machine search procedure, it is necessary to express  $V(w)$  as a function of  $w_{31}$  and use the optimum search program. For this statement (4.5) and the values  $w_{31} = 0.82$ :  $V(w_{31}) = 0.000769 \cdot 0.82^{w_{31}}$ . For this function, the largest upper bound corresponds to the minimum value  $w_{31}(w_{31} = 0.82)$ ;  $V(w_{31}) = 0.000656$ .

From the conditions of partial invariance for dominant weights  $w_{01}$  and  $w_{11}$ :

$$\begin{cases} 0.000656 = 0.01 \cdot n^{-0.3} \cdot f_c^{0.2}; \\ \frac{1}{1} = 0.0017 \cdot f_c^{0.2} \cdot n, \end{cases}$$

we get the values of the optimized parameters:  $f_{c3}^0 = 0.28$  mm/rev;  $n_3^0 = 756.56$  rpm;  $V_3^0 = 237.56$  m/min and the value of machine time  $t_m = 1.17$  min. Thus, as a result of applying the climb-down method, a compromise solution has been found:  $f_{c3}^0 = 0.28$ ;  $n_3^0 = 756.56$ , which provides the greatest approximation of both criteria  $t_m$  and  $f_d$ .

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## 7. OPTIMIZATION OF CUTTING CONDITIONS IN DRILLING

### *7.1. Brief theoretical information*

As a method of mechanical processing, drilling is intended for obtaining blind and through holes in solid, characterized by 14...11 quality, roughness  $R_z$  80... $R_z$  20, and differs:

- 1) variable cutting speed along the length of the cutting edge from  $V = 0 \dots V_{\max}$ ;
- 2) changing rake and clearance angles along the length of the cutting edge;
- 3) the presence of a transverse edge or jumper, which complicates the cutting process;
- 4) difficult chip removal;
- 5) low rigidity of the tool and the technological system as a whole.

When drilling parts from hard-to-cut steels and alloys, both standard and special drills of increased rigidity are used. They are made from high-speed steels DIN 1.3343 (R6M5), DIN 1.3243 (R6M5K5) and hard alloys K30 (VK8), etc. [102-104]. Thus, when machining workpieces made of titanium-based alloys, it is advisable to use four-lands twist drills, which have a greater durability and provide a smaller increase in the hole diameter than conventional two-lands drills (Fig. 22, Fig. 23).

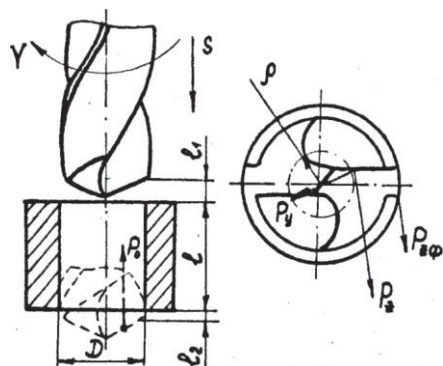


Fig. 22. Drilling pattern

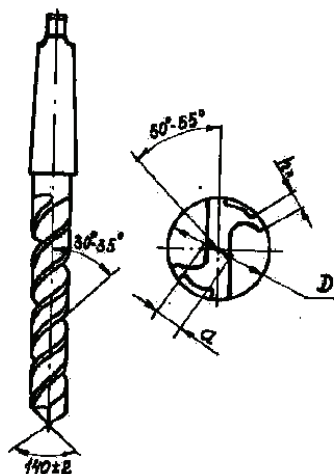


Fig. 23. Four-land drill

In the process of drilling structural materials, the tool is under the action of significant axial compressive forces  $P_0$  and torque  $M_t$ . These factors limit the assigned cutting modes and are the main ones when considering a set of constraints in the optimization problem.

### *Formation of technical constraints*

1. The constraint on the cutting capabilities (tool life), which establishes the relationship between the cutting speed  $V$  and the machinability index  $VT$  (cutting speed corresponding to a certain tool life)  $VT$ .

Using the known dependencies [45, 46]:

$$V = \frac{\pi \cdot D \cdot n}{1000}; \quad V_T = \frac{C_V \cdot K_V \cdot D^{q_v}}{T^m \cdot f_c^{y_v}},$$

where  $C_V, K_V$  – constants that take into account the operating conditions of the tool in the reference and specific versions;  $D$  – hole diameter, mm;  $T$  – tool life period, min;  $f_c$  – feed, mm/rev;  $n$  – rotation frequency, rpm;  $q_v, m, y_v$  – exponents reflecting the influence of diameter, tool life and feed on cutting speed.

We bring this constraint to an explicit form:

$$n \cdot f^{y_v} \leq \frac{318 \cdot D^{(q_v-1)} C_V \cdot K_V}{T^m}. \quad (17)$$

2. The power constraint of the machine  $M_{tm}$ . It binds the torque that occurs during drilling  $M_t$  and affects the spindle of the machine:

$$M_t = 10 \cdot C_m \cdot f^{y_m} \cdot D^{q_m} \cdot K_m;$$
$$M_{tm} = \frac{975 \cdot 10^3 \cdot N_m \cdot \eta}{n},$$

where  $C_m$  – coefficient taking into account the influence of the processing conditions;  $K_m$  – the general correction factor, taking into account the actual processing conditions and depending only on the workpiece materials;  $y_m, q_m$  – indicators of degrees with variables;  $N_m$  – capacity of the main drivers of an electric motor, kW;  $\eta$  – coefficient of performance.

Equating the right parts of the last two dependencies and making the corresponding transformations, we obtain:

$$n \cdot f_c^{y_m} \leq \frac{975 \cdot 10^3 \cdot N_m \cdot \eta}{C_m \cdot D^{q_m} \cdot K_m}. \quad (18)$$

The constraint on  $M_t$  guarantees the integrity of the drill in the presence of stresses in its material.

3. Strength constraint of the machine feed mechanism. To implement the cutting process, the condition must perform:

$$P_a = 10 \cdot C_p \cdot D^{q_p} \cdot f_c^{y_p} \cdot K_p \leq [P_m],$$

where  $P_a$ ,  $[P_m]$  – respectively, the axial and maximum cutting forces allowed by the feed mechanism of the machine  $N$ ;  $C_p$ ,  $q_p$ ,  $y_p$ ,  $K_p$  – determined by reference.

This constraint is explicitly presented below:

$$f_c^{y_p} \leq \frac{[P]}{10 \cdot C_p \cdot D^{q_p} \cdot K_p}. \quad (19)$$

4. The constraint on the strength of the tool. The condition for the drilling strength is expressed by the dependence:

$$\tau_s = \frac{1.73 \cdot M_t}{W} = \frac{1.73 \cdot 10 \cdot C_m \cdot f_c^{y_m} \cdot D^{q_m} \cdot K_p}{0.02 \cdot D^3} \leq \frac{\sigma_b}{K_{sf}},$$

where  $\tau_s$  – the total stress equal to the sum of the normal stress ( $0.73 \tau_s$ ) of the force  $P_a$  and shear stress of  $M_t$ , MPa;  $\sigma_b$  – temporary resistance of the drill material to rupture, MPa;  $K_{sf} = 1.5 \dots 2.0$  – safety factor;  $W$  – the moment of resistance of the drill section,  $\text{mm}^3$ .

After the corresponding transformations, we obtain:

$$f_c^{y_m} \leq \frac{\sigma_b \cdot 0.02 \cdot D^3}{1.73 \cdot 10 \cdot C_m \cdot K_{sf} \cdot D^{q_m} \cdot K_p}. \quad (20)$$

5. The constraint on the rigidity of the cutting tool (Fig. 24).

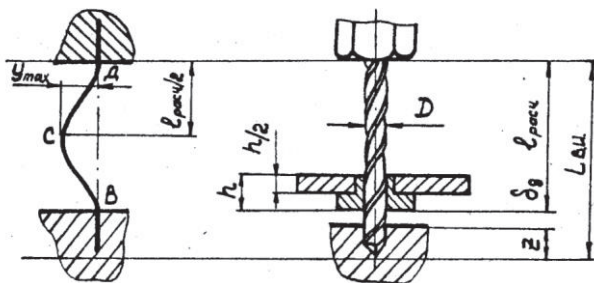


Fig. 24. Scheme of drilling using jig locate

The rigidity of the tool affects the accuracy of processing, while the axial force should not exceed the permissible axial force  $[P_a]$  on the rigidity of the drill:

$$10 \cdot C_p \cdot D^{q_p} \cdot f_c^{y_p} \cdot K_p \leq \frac{K_s \cdot E \cdot I}{L_{od}^2},$$

$K_s$  – stability coefficient,  $K_s \approx 2.46$ ;  $E$  – elastic modulus of the drill material, MPa;  $I = 0.039 \cdot D^4$  – the moment of inertia of the drill,  $\text{mm}^4$ ;  $L_{od}$  – the drill overhang length.

In explicit form, this restriction is written:

$$f_c^{y_p} \leq \frac{K_s \cdot E \cdot I}{L_{od}^2 \cdot 10 \cdot C_p \cdot D^{q_p} \cdot K_p}. \quad (21)$$

The fulfillment of constraint 5 guarantees the integrity of the drill in the event of a possible loss of longitudinal stability.

The constraints imposed by the machine kinematics, establish the relationship of the calculated values of the rotation frequency and feed with the allowable kinematics of the machine [105]. These conditions can be written as the following inequalities:

$$n_{m \min} \leq n \leq n_{m \max}; \quad f_{cm \min} \leq f_c \leq f_{cm \max}. \quad (22)$$

*Setting the objective function.* For a large number of production situations when the values of economic tool life periods are used in the calculations, the least machine time  $t_m$  or specific processing costs  $C_s$  should be chosen as the objective function [68, 75].

$$t_m = \frac{L}{f_c \cdot n} = \frac{l + l_1 + l_2}{f_c \cdot n}; \quad (23)$$

$$C_s = \frac{a_1}{q} \cdot \left( \frac{1 + \frac{a_n}{a_1} \cdot \frac{1}{T}}{n \cdot f_c} \right), \quad (24)$$

where  $L$  – processing length, mm;  $l$  – hole length, mm;  $l_1, l_2$  – the plunge and overrun cutting of the tool values. Linear parameters determined analytically and by reference data:  $l_1 = \frac{D}{2} \text{ctg} \varphi$ ,  $l_2 = 1 \dots 3$ , mm;

$a_1$  – the cost of the machine tool minute of the basic equipment (cent/min);

$q$  – cutting parameter ( $q = \pi D^2 / 4$  – an area of the removed metal);

$a_t = a_1 t_{ch} + a_2 t_{rg} z / (z + 1) + b / (z + 1)$  – tool costs for durability period, cent;

$a_2$  – the cost of machine tools for grinding equipment, cent/min;

$b$  – the cost of a new tool, taking into account transport costs and the implementation of waste, cent;

$t_{ch}$  – tool change time, min;

$t_{gr}$  – tool regrinding time, min;

$z$  – the number of regrinding of the tool until it is completely wearing out.

### *Mathematical model cutting process development*

The mathematical model in the problem of optimizing cutting modes during drilling is jointly the system of inequalities (17) – (22) and the objective function equation (23) or (24). Depending on the method used, the original model (17) – (23), (24) undergoes certain transformations. The statements of the optimization problem that are oriented to the method of linear programming are considered below.

*Linear programming.* The transformation of the original model is carried out by logarithm expressions of constraints (17) – (22) and the objective function (23) and obtain the corresponding linear forms.

After the logarithm of inequality (17), we have (an example of reduction)

$$\ln n + y_v \cdot \ln f_c \leq \ln \left( \frac{318 \cdot C_v \cdot K_v \cdot D^{(q-1)}}{T^m} \right). \quad (25)$$

We introduce the notation  $\ln n = x_1$ ;  $\ln f_c = x_2$ ;  
 $\ln \left( \frac{318 \cdot C_v \cdot K_v \cdot D^{(q-1)}}{T^m} \right) = b_1$  and substitute them into expression (25). As a result of the substitution, we obtain the linear form of the inequality

$$x_1 + y_v \cdot x_2 \leq b_1.$$

Transforming inequalities (18) – (23) in this way, we obtain a system of linear inequalities and a linear function for the case of a drilling operation in the form

$$\left. \begin{array}{l} x_1 + y_v \cdot x_2 \leq b_1 \\ x_1 + y_m \cdot x_2 \leq b_2 \\ y_p \cdot x_2 \leq b_3 \\ y_m \cdot x_2 \leq b_4 \\ y_p \cdot x_2 \leq b_5 \\ x_2 \geq b_6 \\ x_2 \leq b_7 \\ x_1 \geq b_8 \\ x_1 \leq b_9 \\ F_o = (x_1 + x_2) \rightarrow \max \end{array} \right\} A$$

The optimal values  $n_0$  and  $f_0$  are calculated by the formulas

$$n_o = e^{x_1}; \quad f_{c0} = e^{x_2}. \quad (26)$$

The above form of mathematical model A provides a description of the cutting process for the drilling case, regardless of the machine type and processing conditions. If the conditions for performing certain operations change, only the free terms  $b_1, b_2, \dots, b_9$  and the coefficients  $y_l, y_m, y_p$  will be other. To determine the optimal modes using model A, it is necessary to find positive values  $\{x_1, x_2\}$  at which the linear form of the objective function (23) would take on the greatest value.

## ***7.2. Typical task 12. Optimization drilling modes using linear programming***

Linear programming. On a vertical-boring machine 2H135, process a through hole with a diameter  $D = 12H13$  to a length  $l = 55$  mm. The workpiece material – Ti Grade 6 (VT5 titanium-based alloy  $\sigma_b = 900$  MPa); type of workpiece – hot-rolled steel. Tool – twist drill with double sharpening. Geometric parameters:  $2\varphi = 140^\circ$ ;  $\alpha = 12^\circ$ ;  $2\varphi_0 = 90^\circ$ ;  $w = 30^\circ$ ; reverse taper 0.1...0.15 mm per 100 mm of length.

Passport data of the vertical drilling machine 2H135:

- the largest drilling diameter  $D = 35$  mm;
- spindle speed,  $\text{min}^{-1}$ :  $n = 31 \dots 1400$ ;
- spindle feed, mm/rev:  $f_c = 0.1 \dots 1.6$ ;
- the greatest feed force:  $P_{mf} = 15000$  N;
- capacity of the main drive electric motor:  $N_m = 4.5$  kW;
- coefficient of performance:  $\eta = 0.8$ .

### *Formation of constraints system*

1. *The constraint on cutting capabilities.* Using tool life dependence (17) and reference data [87]:  $\{C_v = 2.8; q = 0.7; y_v = 0.6; m = 0.5; K_v = 1\}$ , we get:

$$n \cdot f_c^{0.6} \leq \frac{318 \cdot 12^{-0.3} \cdot 2.8}{12^{0.5}} = 122.11.$$

The recommended tool life period  $T$  for the case of hole machining  $D = 12$  mm with a cemented carbide tool is  $T = 12$  min [87].

2. *The power constraint of the main movement drive of the machine.*

According to [87], the components of inequality (18) assume the following values:  $\{C_m = 60; q_m = 1.9; y_m = 0.8; K_p = 1\}$ . Group VII materials (titanium-based alloys) are characterized by the effect of cutting speed on torque. With this in mind, this constraint takes the form:

$$n^{0.85} \cdot f_c^{0.8} \leq \frac{975 \cdot 10^3 \cdot 4.5 \cdot 0.8 \cdot (1000)^{-0.15}}{(\pi \cdot 12)^{-0.15} \cdot 60 \cdot 12^{1.9}} = 314.31.$$

3. *Strength constraint of the machine feed mechanism.* Given the expression (19) and reference data [87]:  $\{C_p = 850; q_p = 1; y_p = 0.7\}$  this constraint takes the form:

$$f_c^{0.7} \leq \frac{15000}{850 \cdot 12} = 1.47.$$

4. *The constraint on the strength of the tool.* Given the expression (20) and reference data [87]:

$$n^{-0.15} \cdot f_c^{0.8} \leq \frac{900 \cdot 0.02 \cdot 12^{1.25} \cdot 318^{-0.15}}{1.73 \cdot 10 \cdot 60 \cdot 1.5} = 0.108.$$

5. *The constraint on the rigidity of the cutting tool.* Given expression (21) and data [87]:

$$f_c^{0.7} \leq \frac{2.46 \cdot 220000 \cdot 0.039 \cdot 12^4}{120^2 \cdot 12 \cdot 850} = 2.98.$$

6. *Parametric constraints on the passport of the machine* [45]:

$$31 \leq n \leq 1400; \quad 0.1 \leq f_c \leq 1.6.$$

### *The formation of the objective function*

As the objective function, we choose the machine time  $t_m$  spent on drilling a through-hole 55 mm long:

$$t_m = \frac{55 + \left( \frac{12}{2} \cdot \text{ctg} 70 + 3 \right)}{f_c \cdot n} = \frac{60}{f_c \cdot n}.$$

### *Mathematical model*

For the case of through-hole drilling, model  $A$  takes the following form:

$$\left. \begin{array}{l}
 x_1 + 0.6 \cdot x_2 \leq 4.8 \\
 0.85 \cdot x_1 + 0.8 \cdot x_2 \leq 5.75 \\
 0.7 \cdot x_2 \leq 0.39 \\
 -0.15 \cdot x_1 + 0.8 \cdot x_2 \leq -2.23 \\
 0.7 \cdot x_2 \leq 1.09 \\
 x_1 \leq 7.24 \\
 x_1 \geq 3.43 \\
 x_2 \leq 0.47 \\
 x_2 \geq -2.3 \\
 F_0 = x_1 + x_2 \rightarrow \max
 \end{array} \right\} A' .$$

### *Graphical interpretation and definition of optimal cutting modes*

To graphically find  $\{n_0 \text{ and } f_{c0}\}$ , it is necessary to construct a polygon of possible solutions to the system of constraints included in  $A'$ .

In Fig. 25 the direct inequalities of the system  $A'$  are shown and the area of possible solutions  $ABC$  of this system is selected [88-91].

The boundary lines  $AB$ ,  $AC$  and  $BC$  intersecting each other, form a polygon, each of the points inside which satisfies the inequalities of all boundary lines of the system  $A$  involved in its formation. To find the optimal combination of elements, it is necessary to determine at which point of the required  $ABC$  polygon the linear function of two variables  $F_0 = (x_1 + x_2)$  will take the maximum value. To do this, you need to move the line  $F_0$  parallel to itself in the direction from the origin. At the vertex of the polygon (in this case, the triangle)  $C$ , the objective function takes on the greatest value.

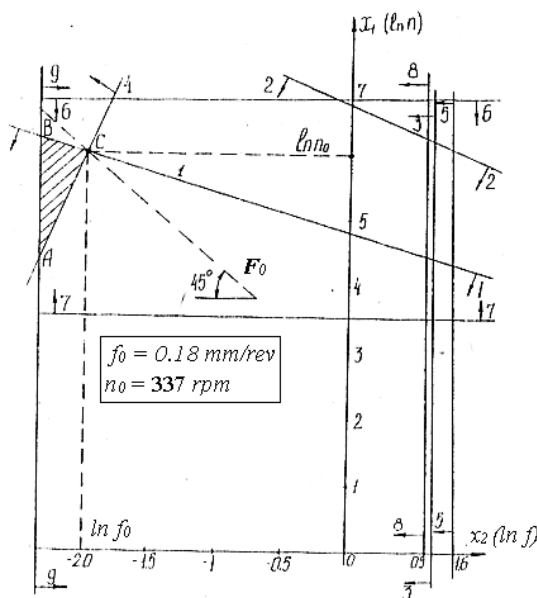


Fig. 25. The scheme of finding the optimal drilling cutting modes

Therefore, vertex  $C$  is the optimum point, and its coordinates  $\{n_0, f_{c0}\}$  are the optimal solution to model  $A'$ .

Thus, the optimal combination of cutting mode elements for the case of drilling a through-hole in a Ti Grade 6 (VT5 titanium alloy) detail is VK8 carbide tool:  $f_{c0} = 0.18 \text{ mm/min}$ ;  $n_0 = 337 \text{ rpm}$ ;  $V_0 = 12.7 \text{ m/min}$ ;

$$f_{cm} = 121.5 \text{ mm/min}; t_m = \frac{55 + 2 + 3}{337 \cdot 0.18} = 0.99 \text{ min.}$$

### 7.3. Typical task 13. Optimization drilling modes using geometric programming

*Geometric programming.* Determine the optimal cutting modes during drilling (source data are borrowed from example 1).

### *Formation of a system of constraints*

Using the obtained polygon of feasible solutions (Fig. 25), we distinguish two active constraints:

1 – constraint on cutting capabilities:

$$n \cdot f_c^{0.6} \leq 122.11.$$

4 – constraint on the strength of the tool:

$$n^{-0.75} \cdot f_c^{0.8} \leq 0.108.$$

We bring the system of active constraints to the standard form in the GP:

$$0.0082 \cdot n \cdot f_c^{0.6} \leq 1;$$

$$9.26 \cdot n^{-0.15} \cdot f_c^{0.8} \leq 1.$$

### *Assignment the objective function*

As the objective function, we consider the specific cost of processing  $C_s$  [62]. According to the data of [1], the components of the dependence (24) take the following values:  $a_1 = 6$  cent/min;  $a_2 = 12$  cent/min;  $t_{ch} = 0.7$  min;  $t_{pa} = 2.0$  min;  $b = 30$  cent;  $z = 15$ ;  $q = \pi \cdot 12^2 / 4 = 113 \text{ mm}^2$ .

Objective function  $C_s$  takes the form:

$$\begin{aligned} C_s &= \frac{0.06}{113} n^{-1} \cdot f_c^{-1} + \frac{0.06 \cdot 0.7 + (0.012 \cdot 2 \cdot 15) / 16 + 0.3 / 16}{113} \cdot T^{-1} \cdot n^{-1} \cdot f_c^{-1} = \\ &= 0.0005 \cdot n^{-1} \cdot f_c^{-1} + 0.007 \cdot T^{-1} \cdot n^{-1} \cdot f_c^{-1}. \end{aligned}$$

Here  $T$  is determined from the extended Taylor equation [45]:

$$T = \frac{(C_v \cdot K_v)^{1/m} \cdot D^{q/m}}{f_c^{y/m} \cdot V^{1/m}} = \frac{(2 \cdot 8 \cdot 1)^{1/0.5} \cdot D^{0.7/0.5}}{f_c^{0.6/0.5} \cdot V^{1/0.5}} = 254.17 \cdot f_c^{-1.2} \cdot V^{-2}.$$

After substituting the expression for  $T$  into the equation of the objective function, we obtain:

$$C_s = 0.0005 \cdot n^{-1} \cdot f_c^{-1} + 3.91 \cdot 10^{-9} \cdot f_c^{0.2} \cdot n.$$

### *Mathematical model*

A direct statement of the GP problem during drilling is represented as follows:

Minimize:

$$g_o(f_c, n) = C_s = 0.0005 \cdot n^{-1} \cdot f_c^{-1} + 3.91 \cdot 10^{-9} \cdot n \cdot f_c^{0.2},$$

under constraints:

$$0.0082 \cdot n \cdot f_c^{0.6} \leq 1;$$

$$9.26 \cdot n^{-0.15} \cdot f_c^{0.8} \leq 1.$$

This task is the GP task of the first degree of difficulty. To solve it, we use the Wild method [72] and represent it as a zero-degree GP problem by discarding the second term of the objective function associated with the cost of tool changing.

Direct GP statement of zero degrees of difficulty:

Minimize:

$$C_s^1 = 0.0005 \cdot n^{-1} f_c^{-1} + 3.91 \cdot 10^{-9} \cdot n \cdot f_c^{0.2}, \quad (27)$$

under constraints:

$$0.0082 \cdot n \cdot f_c^{0.6} \leq 1; \quad (28)$$

$$9.26 \cdot n^{-0.15} \cdot f_c^{0.8} \leq 1.$$

The dual GP statement of zero degrees of difficulty:

maximize:

$$V(w) = \left( \frac{C_{01}}{w_{01}} \right)^{w_{01}} \cdot C_{11}^{w_{11}} \cdot C_{21}^{w_{21}},$$

under the constraints:

$$\begin{aligned} w_{01} &= 1; \\ -w_{01} + w_{11} - 0.15w_{21} &= 0; \\ -w_{01} + 0.6w_{11} + 0.8w_{21} &= 0. \end{aligned}$$

The solution of the last system of linear equations allows us to uniquely determine the dual weights  $w_i$ :

$$\{w_{01} = 1; w_{11} = 1.7; w_{21} = 0.45\};$$

The value of the objective function is:

$$V(w) = 0.005 \cdot 0.0082^{1.07} \cdot 9.26^{0.45} = 8.03 \cdot 10^{-6}, \text{ EUR/mm}^3.$$

From the conditions of invariance, we determine the optimal values of the cutting modes:

$$\begin{aligned} 0.0082 \cdot n \cdot f_c^{0.6} &= 1; \\ 9.26 \cdot n^{-0.15} f_c^{0.8} &= 1. \end{aligned}$$

As a result of solving the last system of equations, we obtain:

$$n_0 = 337 \text{ rpm}; f_{c0} = 0.18 \text{ mm/rev}; V_0 = 12.7 \text{ m/min}.$$

By the partial invariance method [68], we carry out the following sequence of procedures:

1. We will formulate a model of the GP problem of the first degree of difficulty by introducing an additional term related to the cost of tool changing in the objective function (27):

$$C_s^H = 0.0005 \cdot n^{-1} \cdot f_c^{-1} + 3.91 \cdot 10^{-9} \cdot f_c^{0.2} \cdot n. \quad (29)$$

The constraint for this option is stated above (28).

2. We define the lower bound of the objective function  $C_s''$  in the statement (28), (29). In this case, the optimal values of the variables obtained as a result of solving the original problem of zero degrees of difficulty should be used. The cost of processing in this case:

$$\begin{aligned} C_s'' &= 0.0005 \cdot 337^{-1} \cdot 0.18^{-1} + 3.91 \cdot 10^{-9} \cdot 337 \cdot 0.18^{0.2} = \\ &= 8.24 \cdot 10^{-6} + 0.935 \cdot 10^{-6} = 9.18 \cdot 10^{-6}. \end{aligned}$$

3. We calculate the basic weights of the members of the objective function by dividing the components of  $C_s$  by their sum:

$$w_{01}^I = \frac{8.24}{9.18} = 0.898; \quad w_{02}^I = \frac{0.935}{9.18} = 0.102.$$

The dual weights of the constraints  $\{w_1, w_2\}$  remain unchanged. A new set of dual scales is

$$w_{01} = 0.898; \quad w_{02} = 0.102; \quad w_{11} = 1.07; \quad w_{21} = 0.45.$$

4. We compose a system of linear equations in the GP dual statement, including the conditions of orthogonality and normalization using the system (28), (29).

$$\begin{aligned} f_c : -w_{01} + 0.2w_{02} + 0.6w_{11} + 0.8w_{21} &= 0; \\ n : -w_{01} + w_{02} + w_{11} - 0.15w_{21} &= 0; \\ w_{01} + w_{02} &= 1. \end{aligned} \tag{30}$$

5. We choose the dominant terms in the system (30), which does not have a unique solution. These should include the most significant members in the optimal project, and their number should exceed the number of project variables ( $f_c, n$ ) by one unit. For this example, such members will be the first ( $w_{01} = 0.898$ ), the third ( $w_{11} = 1.07$ ) and the fourth ( $w_{21} = 0.45$ ).

6. We solve the system of linear equations (30) to the dominant dual variables, i.e. variables corresponding to dominant members. In the case under consideration, linear dependences on  $w_{02}$  are obtained:

$$w_{01} = 1 - w_{02}; w_{11} = 1,068 - 2w_{02}; w_{21} = 0,45w_{02}.$$

For all  $w_i$  to be positive,  $w_{02}$  it must be less than 0.5338. For this problem of the first degree of difficulty (6.20), (6.19) it is easy to calculate the dual function for various values of the only redundant dual variable (in our example,  $w_{02}$  – the weight of the objective function term associated with the tool change). The same dual function can be calculated for the values of the weights  $w_i$  giving the largest lower bound, and thereby determine the minimum of the “direct” function in the “direct” formulation.

7. Let us express the dual function in a general form as a function of the dual weight  $w_{02}$ :

$$\begin{aligned} V(w_{02}) &= \frac{\left(\frac{0.0005}{1-w_{02}}\right) \cdot \left(\frac{1-w_{02}}{0.0005}\right) \cdot 3.91 \cdot 0.0082^{1.068} \cdot 9.26^{0.45}}{w_{02}^{w_{02}} \cdot 0.0082^{2w_{02}} \cdot 10^{9w_{02}}} = \\ &= \frac{8.04 \cdot 10^{-6} \cdot (1-w_{02})^{w_{02}-1} \cdot 0.12^{w_{02}}}{w_{02}^{w_{02}}}. \end{aligned} \quad (31)$$

8. We use the dichotomy method [38, 68] to determine the maximum of the function  $V(w_{02})$ . Using the developed program for optimizing unimodal functions, the following result was obtained:

$$w_{02} = 0.019; V(w_{02}) = C_s = 9.003 \cdot 10^{-6} \text{ EUR/mm}^3.$$

9. Determine the optimal cutting modes that minimize the specific processing cost, based on the conditions of invariance:

$$\begin{aligned} 0.005 \cdot n^{-1} \cdot f_c^{-1} &= 9.003 \cdot 10^{-6} \cdot 0.8981; \\ 0.0032 \cdot n \cdot f_c^{0.6} &= 1. \end{aligned} \quad (32)$$

As a result of solving the system of nonlinear equations (32), we obtain:

$$\begin{aligned} n_0 &= 337.52 \text{ rpm}; f_{c0} = 0.183 \text{ mm/rev}; V_0 = 12.72 \text{ m/min}; \\ f_m &= 61.83 \text{ mm/min}; C_{s0} = 9.003 \cdot 10^{-6} \text{ EUR/mm}^3 \end{aligned}$$

#### **7.4. Typical task 14. Optimization drilling modes using method of Lagrange multipliers**

The method of Lagrange multipliers. Determine the optimal cutting modes during drilling (source data are borrowed from example 1).

Let us consider the use of the Lagrange multiplier method for solving the optimization problems formulated above.

*Drilling. The objective function* and constraints on cutting tool capabilities and tool strength respectively are of the form:

$$\begin{aligned} C &= 0.005 \cdot n^{-1} \cdot f_c^{-1} + 3.91 \cdot 10^{-9} \cdot n \cdot f_c^{0.2} = \alpha_1 \cdot n^{-1} \cdot f_c^{-1} + \alpha_2 \cdot n \cdot f_c^{0.2} \\ n \cdot f_c^{0.2} &\leq 122.11 = \beta_1; \\ n^{-0.15} \cdot f_c^{0.8} &\leq 0.108 = \beta_2. \end{aligned}$$

We compose the Lagrange function:

$$L = C + \lambda_1 \cdot (\beta_1 - n \cdot f_c^{0.2}) + \lambda_2 \cdot (\beta_2 - n^{-0.15} \cdot f_c^{0.8}),$$

where  $\lambda_1 \lambda_2$  – Lagrange multipliers.

Next, we have

$$\begin{cases} \frac{\partial L}{\partial n} = -\alpha_1 \cdot n^{-2} \cdot f_c^{-1} + \alpha_2 \cdot f_c^{0.2} - \lambda_1 \cdot f_c^{0.6} + 0.15 \cdot \lambda_2 \cdot n^{-1.15} \cdot f_c^{0.8} = 0; \\ \frac{\partial L}{\partial f} = -\alpha_1 \cdot n^{-1} \cdot f_c^{-2} + 0.2 \cdot \alpha_2 \cdot n \cdot f_c^{-0.8} - 0.6 \cdot \lambda_1 \cdot f_c^{-0.4} - 0.8 \cdot \lambda_2 \cdot n^{-0.15} \cdot f_c^{-0.2} = 0; \\ \frac{\partial L}{\partial \lambda_1} = \beta_1 - n \cdot f_c^{0.6} = 0; \\ \frac{\partial L}{\partial \lambda_2} = \beta_2 - n^{-0.15} \cdot f_c^{0.8} = 0. \end{cases}$$

Let  $\lambda_1 \neq 0$ ;  $\lambda_2 = 0$ . Then we get:

$$\frac{-\alpha_1}{n \cdot f_c} + \alpha_2 \cdot n \cdot f_c^{0.2} - \lambda_1 \cdot \beta_1 = 0;$$

$$\frac{-\alpha_1}{n \cdot f_c} + 0.2 \cdot \alpha_2 \cdot n \cdot f_c^{0.2} - 0.6 \cdot \lambda_1 \cdot \beta_1 = 0;$$

$$\frac{-0.4\alpha_1}{n \cdot f_c} + 0.4\alpha_2 \cdot n \cdot f_c^{0.2} = 0 \rightarrow \begin{cases} \alpha_2 \cdot n^2 \cdot f_c^{1.2} = \alpha_1 \\ n \cdot f_c^{0.6} = \beta_1 \end{cases} \rightarrow \begin{cases} f_c^{0.84} = \frac{\alpha_1}{\alpha_2 \cdot \beta_1^2} \\ n = \beta_1 \cdot f_c^{-0.6} \end{cases}.$$

Thus, one of the possible solutions has the form:

$$\begin{cases} f_c = 8.58^{1.19} = 12.91; \\ n = 122.11 \cdot 12.91^{-0.6} = 26.31. \end{cases}$$

We check this solution for admissibility. To do this, substitute the found value in the second constraint:

$$26.31^{-0.15} \cdot 12.91^{0.8} = 4.74 > 0.108,$$

i.e. the constraint is not fulfilled and the solution is not in the range of acceptable values.

Let  $\lambda_1 = 0$ ;  $\lambda_2 \neq 0$ . Then we get:

$$\begin{cases} \frac{-\alpha_1}{n \cdot f_c} + \alpha_2 \cdot n \cdot f_c^{0.2} + 0.15 \cdot \lambda_2 \beta_2 = 0; \\ \frac{-\alpha_1}{n \cdot f_c} + 0.2 \cdot \alpha_2 \cdot n \cdot f_c^{0.2} - 0.8 \cdot \lambda_2 \cdot \beta_2 = 0; \end{cases}$$

$$-9.5 \cdot \alpha_1 \cdot n^{-1} \cdot f_c^{-1} + 8.31 \cdot \alpha_2 \cdot n \cdot f_c^{0.2} = 0 \rightarrow \left\{ \begin{array}{l} 8.31 \cdot \alpha_2 \cdot n^2 \cdot f_c^{1.2} = 9.5 \cdot \alpha_1 \\ n^{-0.15} \cdot f_c^{0.8} = \beta_2 \end{array} \right\} \rightarrow$$

$$\rightarrow \left\{ \begin{array}{l} n = \frac{1.07 \cdot \left( \frac{\alpha_1}{\alpha_2} \right)^{0.5}}{f_c^{0.6}}; \\ f_c^{0.89} = \frac{\beta_2}{1.07^{-0.15} \cdot \left( \frac{\alpha_1}{\alpha_2} \right)^{-0.075}} \end{array} \right\}.$$

The second possible solution has the form

$$f_c = \left( \frac{0.108}{1.07^{-0.15} \cdot \left( \frac{0.0005}{3.91 \cdot 10^{-9}} \right)^{-0.075}} \right)^{\frac{1}{0.89}} = \left( \frac{0.26}{0.99} \right)^{1.12} = 0.22;$$

$$n = \frac{389.78}{0.22^{0.6}} = 966.86.$$

Substitute the found values into the first constraint:

$$966.86 \cdot 0.22^{1.12} = 177.36 > 122.11,$$

i.e. the constraint is not met.

Let  $\lambda_1 \neq 0$ ;  $\lambda_2 \neq 0$ . Then the solution of the general problem is the solution of the system of equations

$$\left\{ \begin{array}{l} n \cdot f_c^{0.6} = \beta_1 \\ n^{-0.15} f_c^{0.8} = \beta_2 \end{array} \right\} \rightarrow \left\{ \begin{array}{l} n = \frac{\beta_1}{f_c^{0.6}} \\ f_c^{0.89} = \frac{\beta_2}{\beta_1^{-0.15}} \end{array} \right.$$

The third possible solution is

$$f_c = \left( \frac{0.108}{122.11^{-0.15}} \right)^{1.12} = 0.185;$$

$$n = \frac{122.11}{0.185^{0.6}} = 336.08.$$

This solution satisfies both constraints and is the desired solution to the problem

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## 8. OPTIMIZATION OF CUTTING CONDITIONS IN MILLING

### *8.1. Brief theoretical information*

Milling is a method of multi-blade machining of groove planes, shaped surfaces, bodies of revolution, as well as the manufacture of splines and cutting the workpiece, which allows us to obtain a surface roughness of  $R_z$  40 ...  $R_a$  2.5 and a quality grade of 12 ... 9. The features of the cutting process during milling include:

- the simultaneous presence in the process of cutting several teeth. The larger this number, the lower the intensity of the vibrations during cutting;
- cyclic stresses on the tooth in the mode: load – rest;
- periodically repeated the tooth entrance into the metal, leading to impact loads on the cutting edges, as well as in the presence of a rounding radius, the occurrence of a certain sliding period without cutting;
- variability of the cutting edge load for one cutting cycle, due to the variable size of the area of the cut layer.

There are milling processes performed by the periphery- and face-milling cutter. The latter include schemes of symmetrical and asymmetric milling, as well as counter (when the movement of working teeth milling cutter during its rotation is directed against the direction of feed) and along (with the coincidence of directions) milling.

Face milling of difficult-to-machine materials is the most of rational to conduct according to the scheme of an incomplete asymmetrical milling cutter (Fig. 26).

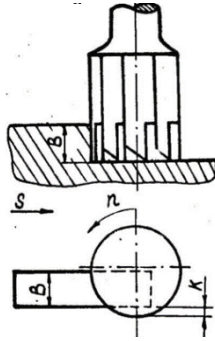


Fig. 26. Scheme of face milling

The displacement  $K$  of the cutter axis relative to the workpiece when milling hard-to-cut steels and alloys should be within  $0.05 \dots 0.1$  cutter diameter  $D_m$ . When  $K > 0.1 \cdot D_m$  cutter life sharply decreases.

Tools used for processing hard-to-cut materials are equipped with multi-faceted or brazed inserts made of high-speed steels and hard alloys.

The diameter of the cutter should be chosen so that the ratio of the width of the milled surface to the diameter is  $0.6 \dots 0.7$ . The method for choosing cutters for cutting hard-to-cut materials is given in [8, 9, 23].

The milling process takes place in the condition of a specific technological system and is described by certain permissible characteristics of cutting capacity, levels of tool loads, tool life and reliability of operation. The composition of constraints on possible sets of cutting modes includes the following set.

### *Technical constraints during milling*

1. Constraint on the cutting capabilities of the tool:

$$n \cdot f_c^{y_v} \leq \frac{318 \cdot C_v \cdot D_m^{q_v-1} \cdot K_v}{T^m \cdot d_c^{x_v} \cdot z^{u_v} \cdot B^{r_v}}, \quad (33)$$

where  $D_m$  – mill cutter diameter, mm;  $z$  – cutter number of teeth;  $B$  – the width of the milling, mm

2. Machine capacity constraint:

$$n \cdot f_c^{y_p} \leq \frac{975 \cdot 10^3 \cdot N_m \cdot \eta \cdot K_{cz}}{C_p \cdot d_c^{x_p} \cdot z^{u_p} \cdot B^{r_p} \cdot K_p \cdot D_m^{1-q_p}}. \quad (34)$$

3. Strength constraint of the machine feed mechanism. To implement the milling process (Fig. 27), the condition must be carried out:

$$P_f \leq [P_m],$$

where  $P_f$  – force that the machine feed mechanism overcomes. In the general case:

$$P_f = P_c + F,$$

where  $P_c$  – a force generated in the cutting process;  $F$  – friction force in the machine guides.

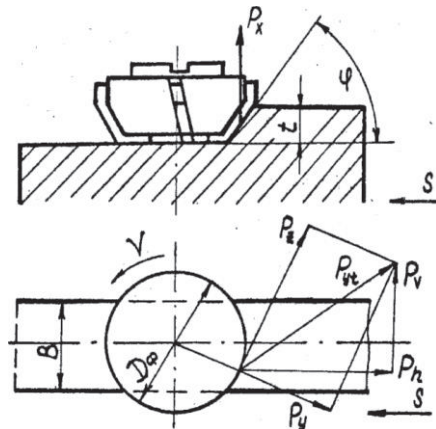


Fig. 27. Milling force system

For the milling process, the force  $P_f$  is determined by the following dependence (taking into account relations [16] and the value of the friction coefficient  $f_{fr} = 0.1$ ):

$$P_f = P_c + f_{fr} (P_z + P_y + P_x) = 0.7 \cdot P_z + 0.1 \cdot (P_z + 0.4 \cdot P_z + 0.5 P_z) = 0.89 \cdot P_z.$$

Explicitly, the feed constraint takes the form:

$$f_c^{y_p} \leq \frac{P_f}{C_p \cdot d_c^{x_p} \cdot z^{u_p} \cdot B^{r_p} \cdot K_p \cdot D_m^{1-q_p}}. \quad (35)$$

Constraints on the strength of the cutter, the strength of the cemented-carbide tip, the rigidity of the cutting tool and the workpiece, as a rule, are not taken into account when solving optimization problems [92].

4. Parametric constraints imposed by the kinematics of the machine:

$$n_{m.\min} \leq n \leq n_{m.\max}; f_{m.\min} \leq f \leq f_{m.\max}. \quad (36)$$

### *Objective functions during milling*

1. The main time in cut  $t_c$  for an operation performed on milling machines is calculated by the formula

$$t_c = \frac{L}{f_m} i = \frac{l + l_1 + l_2}{f_z \cdot z \cdot n} i, \quad (37)$$

where  $L$  – length of the path movement by the tool in the feed direction, mm;

$l$  – length of the treated surface, mm;

$l_1$  – value for tool cutting-in and overrun, mm;

$l_2$  – additional length for taking test chips. Depending on the mill cutter size –  $l_2 = 5 \dots 10$  mm;

$f_m$  – feed cutter minute, mm/min;

$i$  – number of passes.

2. The variable part of the cost price  $C$ , depending on the cutting modes, is written as:

$$C = A \cdot t_m + (A \cdot t_{ch} + A') \cdot \frac{t_m}{T}, \quad (38)$$

where  $A$  – the cost of machine time, cent/min;

$A'$  – cost of the tool, reduced to one period of tool life (depends on the type of tool), cent/ min;

$t_m \approx t_c$  – machine time, min;

$t_{ch}$  – tool change time, min.

The tool change time during milling depends on the number of teeth of the cutter [93-96] and the method of grinding the back surface (without backing, one and two-time backing) and can be written as:

$$t_{ch} = t_i + t_g \cdot Z,$$

where  $t_i$  – innovation time (to mount the cutter and the machine adjustment), min;

$t_g$  – grinding time for one tooth, min.

### *Mathematical model of the milling process*

The mathematical model in the problem of optimizing cutting modes during milling is the combination of the system of inequalities (33)...(36) and the equations of the objective function (6.28) or (6.29).

When using the linear programming method, the desired process model is:

$$\left. \begin{array}{l} x_1 + y_v \cdot x_2 \leq b_1 \\ x_1 + y_p \cdot x_2 \leq b_2 \\ y_p \cdot x_2 \leq b_3 \\ x_2 \geq b_4 \\ x_2 \leq b_5 \\ x_1 \geq b_6 \\ x_1 \leq b_7 \\ F_o = x_1 + x_2 \rightarrow \max \end{array} \right\} B.$$

The optimal values of  $n_0$  and  $f_{z0}$  are calculated by formulas (26).

## 8.2. Typical task 15. Optimization milling modes using linear programming

Linear programming. On a vertical milling machine, process a base type part characterized by the following dimensions:  $B = 100$  mm;  $l = 300$  mm;  $R_z = 40$   $\mu$ m; part material heat-resistant steel DIN X6 Cr Ni Ti 18 (12X18H9T ( $\sigma_b = 660$  MPa); workpiece – forging; allowance  $h = 4$  mm. Machine vertically milling 6R13F301. Passport data: spindle rotation frequency,  $\text{min}^{-1}$ :  $n = 40 \dots 2000$  (step less regulation); table feed, mm/min:  $f_m = 10 \dots 2000$  (step less regulation); the greatest feed force:  $[P_f] = 7550$  N; the capacity of the electric motor of the main drive  $N_m = 7.5$  kW; coefficient of performance  $\eta = 0.8$ . End mill,  $D_m = 150$  mm; the number of teeth  $z = 6$ ;  $\varphi = 60^\circ$ ; material of cutting tip – VK8 hard alloy. Mounted the mill on the limb. Mounted the detail on a table with fastening with bolts and slats with a simple alignment. Detail weight – 5 kg. Small batch production.

### *Formation of a system of constraints*

1. Constraints on cutting capabilities. Using the dependence (33) and reference data [6], we represent in the form of inequality:

$$V = \frac{108 \cdot D_m^{0.2}}{T^{0.32} \cdot d_c^{0.06} \cdot f_z^{0.3} \cdot B^{0.2}}.$$

We form the constraint explicitly:

$$n \cdot f_z^{0.3} \leq \frac{318 \cdot 108 \cdot 150^{0.2}}{150 \cdot 180^{0.32} \cdot 4^{0.06} 100^{0.2}} = 43.37.$$

The tool life period  $T$  (average value) can be assigned according to the reference manual [30] depending on the type of mill cutter and its diameter. For this case,  $T = 180$  min.

2. The drive capacity constraint of the machine main movement. According to [30], the components of inequality (34) take the values:

$$\{C_p = 218; x_p = 0,92; y_p = 0,78; u_p = 1; q_p = 1,15\},$$

in addition, the constraint takes the form:

$$n \cdot f_z^{0.78} \leq \frac{975 \cdot 10^3 \cdot 7.5 \cdot 0.8 \cdot 150^{1.15} \cdot 2}{218 \cdot 4^{0.92} \cdot 100 \cdot 6 \cdot 150} = 52.98.$$

3. Strength constraint of the feed mechanism. Taking into account (35) and data [45]:

$$f_z^{0.78} \leq \frac{7550 \cdot 150^{1.15}}{0.89 \cdot 10 \cdot 218 \cdot 4^{0.92} \cdot 100 \cdot 6} = 0.58.$$

4. Parametric constraints on the machine passport:

$$\begin{aligned} 40 &\leq n \leq 2000; \\ 0.00083 &\leq f_z \leq 8.33. \end{aligned}$$

### *The formation of the objective function*

As the objective function, we choose the machine time spent on milling the plane ( $l = 300$  mm):

$$t_m = \frac{l + \left[ 0.5 \cdot \left( D_m - \sqrt{D_m^2 - B^2} \right) + 5 \right] + l_2}{f_m} \cdot i.$$

After substituting the data of example 4 and [19], we obtain

$$t_m = \frac{300 + \left[ 0.5 \cdot \left( 150 - \sqrt{150^2 - 100^2} \right) + 5 \right] + 10}{6 \cdot f_z \cdot n} = \frac{55.68}{f_z \cdot n}.$$

### *Mathematical model*

For the case of face milling, model B will take the following form:

$$\left. \begin{array}{l} x_1 + 0.3 \cdot x_2 \leq 3.77 \\ x_1 + 0.78 \cdot x_2 \leq 3.97 \\ 0.78 \cdot x_2 \leq -0.54 \\ x_1 \leq 7.6 \\ x_1 \geq 3.69 \\ x_2 \leq 2.12 \\ x_2 \geq -7.9 \\ F_o = x_1 + x_2 \rightarrow \max \end{array} \right\} B'.$$

### *Graphical interpretation and determination of optimum cutting modes*

In Fig. 28, double logarithmic scales depict straight lines describing the inequalities of the system  $B'$  the area of possible solutions  $\{A'B'C'D'\}$  is selected. The linear function  $F_o = x_1 + x_2$  will take a maximum at point  $C'$ , and the coordinates of this point,  $\{f_{z0}; n_0\}$  are the optimal solution to the system  $B'$ . They are  $f_{z0} = 0.497$  mm/tooth;  $n_0 = 53.3$  rpm;  $V_0 = 25.11$  m/min;  $t_{m0} = 2.1$  min.

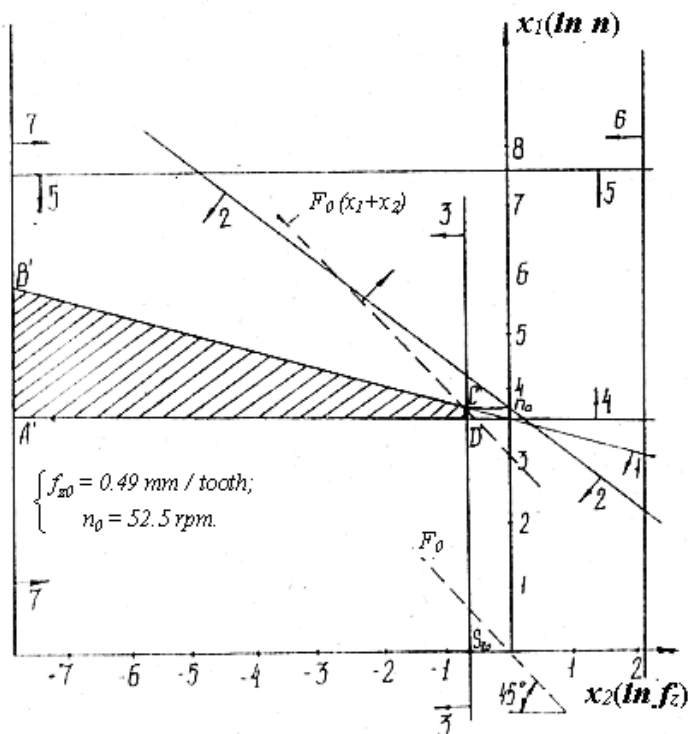


Fig. 28. The scheme of finding the optimal milling modes

### 8.3. Typical task 16. Optimization milling modes using geometric programming

Determine the optimal cutting modes for face milling (source data borrowed from example 4).

#### *Formation of a system of constraints*

Using the result in the previous example, we select two active constraints that determine the optimal value of cutting modes:

Cutting capacity constraint:

$$n \cdot f_z^{0.3} \leq 43.37.$$

Strength constraint of the machine feed mechanism:

$$f_z^{0.78} \leq 0.58.$$

We bring these constraints to the standard form in the GP:

$$\begin{aligned} 0.026 \cdot n \cdot f_z^{0.3} &\leq 1; \\ 1.724 \cdot f_z^{0.78} &\leq 1. \end{aligned}$$

### *Assignment the objective function*

As an objective function, we consider a variable part of the cost price, depending on the cutting modes. According to the data of [1], the components of the dependence (38) take the following values:  $A = 3.82$  cent/min, [86];  $t_i = 1.35$  min [30];  $t_g = 2$  min [86];  $A' = 0.63$  cent/min [86].

The objective function (38) when substituting the dependences  $T = F(f_z, n)$  and  $t_m = F(f_z, n)$  takes the form:

$$\begin{aligned} C &= \frac{3.82 \cdot 55.68}{f_z \cdot n} + [3.82 \cdot (1.35 + 2 \cdot 6) + 0.63] \cdot \frac{55.68 \cdot n^{3.13} \cdot f_z^{0.94}}{0.24 \cdot 10^8 \cdot f_z \cdot n} = \\ &= 212.7 \cdot f_z^{-1} \cdot n^{-1} + 1.19 \cdot 10^{-4} \cdot f_z^{-0.06} \cdot n^{2.13}. \end{aligned}$$

### *Mathematical model*

A direct statement of the GP problem in face milling is presented in the following form:

Minimize:

$$g_0(n, f_z) = 212.7 \cdot f_z^{-1} \cdot n^{-1} + 1.19 \cdot 10^{-4} \cdot f_z^{-0.06} \cdot n^{2.13}, \quad (39)$$

under the constraints:

$$0.026 \cdot n \cdot f_z^{0.3} \leq 1; \quad (40)$$

$$1.724 \cdot f_z^{0.78} \leq 1. \quad (41)$$

This task has a first degree of difficulty. We use the Wild technique and, similarly to Example 2, we solve the optimization problem.

First, we transform the statement (39) ... (41). As a GP problem of zero degrees of difficulty by discarding the term associated with the cost of the tool.

Minimize:

$$C^J = 212.7 \cdot f_z^{-1} \cdot n^{-1},$$

with constraints:

$$0.026 \cdot n \cdot f_z^{0.3} \leq 1;$$

$$1.724 \cdot f_z^{0.78} \leq 1.$$

Corresponding dual form:

Maximize:

$$V(w) = \left( \frac{C_{01}}{w_{01}} \right)^{w_{01}} \cdot C_{11}^{w_{11}} \cdot C_{21}^{w_{21}},$$

under the constraints:

$$w_{01} = 1;$$

$$n : -w_{01} + w_{11} = 0; \quad (42)$$

$$f_z : -w_{01} + 0.3 \cdot w_{11} + 0.78 \cdot w_{21} = 0.$$

The value of dual weights  $w_i$ :

$$\{w_{01} = 1; w_{11} = 1; w_{21} = 0.9\}.$$

The value of the objective function is:

$$V(w) = 212.7 \cdot 0.023 \cdot 1.724^{0.9} = 8.02 \text{ cent.}$$

From the conditions of invariance, we determine the values of the best cutting modes:

$$\left\{ \begin{array}{l} f_{z0} = 0.497 \frac{\text{mm}}{\text{tooth}}; n_0 = 53.3 \text{ min}^{-1}; V_0 = 25.11 \frac{\text{m}}{\text{min}}; \\ C_0^I = 8.02 \text{ cent.} \end{array} \right\}$$

We form the GP model of the first degree of difficulty:

$$C^{II} = 212.7 \cdot f_z^{-1} \cdot n^{-1} + 1.19 \cdot 10^{-4} \cdot f_z^{0.06} \cdot n^{2.13}. \quad (43)$$

The constraints on this option coincide with those previously calculated as GP problems of the zero degrees of difficulty (40), (41).

We define the lower bound of the objective function  $C^{II}$  in the statement (42), (43):

$$C^{III} = 212.7 \cdot 0.497^{-1} \cdot 53.3^{-1} + 1.19 \cdot 10^{-4} \cdot 0.497^{-0.06} \cdot 53.3^{2.13} = 8.61 \text{ cent.}$$

We calculate the main weights of the terms of the objective function:

$$w_{01}^I = \frac{8.02}{8.61} = 0.93; w_{02}^I = \frac{0.59}{8.61} = 0.07.$$

A new set of dual weights is:

$$\{w_{01} = 0.93; w_{02} = 0.93; w_{11} = 1; w_{21} = 0.9\}.$$

We form the conditions of orthogonality and normalization in the statement (42), (43):

$$\begin{array}{l} n: -w_{01} + 2.13 \cdot w_{02} + w_{11} = 0; \\ f: -w_{01} + 0.06 \cdot w_{02} + 0.3 \cdot w_{11} + 0.78 \cdot w_{21} = 0; \\ w_{01} + w_{02} = 1. \end{array} \quad (44)$$

We choose the dominant terms in system (44). These include the first  $\{w_{01} = 0.93\}$ ; third  $\{w_{11} = 1.0\}$ ; fourth  $\{w_{21} = 0.9\}$ . Imagine:

$$\{w_{01} = 0.93; w_{02} = 0.93; w_{11} = 1; w_{21} = 0.9\}.$$

For all  $w_i$  to be positive, the weight  $w_{02}$  must be at least 0.32. We express the dual function in a general form as a function of the dual weight  $w_{02}$ :

$$\begin{aligned} V(w_{02}) &= \frac{\left(\frac{212.7}{1-w_{02}}\right) \cdot \left(\frac{1-w_{02}}{212.7}\right)^{w_{02}} \cdot 1.19^{w_{02}} \cdot 0.023 \cdot 1.724^{0.9}}{w_{02}^{w_{02}} \cdot 0.023^{3.13 \cdot w_{02}} \cdot 10^{4 \cdot w_{02}}} = \\ &= 7.99 - (1-w_{02})^{w_{02}-1} \cdot \left(\frac{0.08}{w_{02}}\right)^{w_{02}}. \end{aligned} \quad (45)$$

We use the dichotomy method to determine the optimal values of the weights of the terms and the corresponding value of the objective function:

$$\{w_{02} = 0.099; V(w_{02}) = C_0 = 8.59 \text{ cent}\}.$$

We determine the optimal cutting modes that minimize the variable part of the cost price, based on the conditions of invariance:

$$\begin{aligned} 212.7 \cdot n^{-1} \cdot f_z^{-1} &= 0.93 \cdot 8.59; \\ 1.724 \cdot f_z^{0.78} &= 1. \end{aligned} \quad (46)$$

As a result of solving system (6.37), we obtain:

$$\begin{aligned} n_0 &= 53.57 \text{ min}^{-1}; f_{z0} = 0.497 \text{ mm / tooth}; \\ V_0 &= 25.25 \text{ m / min}; f_{m0} = 159.75 \text{ mm / min}; \\ C_0 &= 8.59 \text{ cent}. \end{aligned}$$

#### 8.4. Typical task 17. Optimization milling modes using method of Lagrange multipliers

The objective function and constraints are of the form:

$$C = 212.7 \times f_z^{-1} n^{-1} + 1.19 \cdot 10^{-4} \cdot f_z^{-0.06} n^{2.13} = \alpha_1 \times f_z^{-1} n^{-1} + \alpha_2 \cdot f_z^{-0.06} n^{2.13};$$

$$n f_z^{0.3} \leq 43.47 = \beta_1; \quad - \text{tool cutting capabilities}$$

$$f_z^{0.78} \leq 0.58 = \beta_2. \quad - \text{feed mechanism strenght}$$

We compose the Lagrange function:

$$L = C + \lambda_1 \cdot (\beta_1 - n f_z^{0.3}) + \lambda_2 \cdot (\beta_2 - f_z^{0.78}).$$

Take private derivatives:

$$\frac{\partial L}{\partial n} = -\alpha_1 \cdot f_z^{-1} \cdot n^{-2} + 2.13 \cdot \alpha_2 \cdot f_z^{-0.06} \cdot n^{2.13} - \lambda_1 \cdot f_z^{0.3} = 0;$$

$$\frac{\partial L}{\partial f} = -\alpha_1 \cdot f_z^{-2} \cdot n - 0.06 \cdot \alpha_2 \cdot f_z^{-1.06} \cdot n^{2.13} - 0.3 \cdot \lambda_1 \cdot n \cdot f_z^{-0.7} -$$

$$- 0.78 \cdot \lambda_2 \cdot f_z^{-0.22} = 0;$$

$$\frac{\partial L}{\partial \lambda_1} = \beta_1 - n \cdot f_z^{0.3};$$

$$\frac{\partial L}{\partial \lambda_2} = \beta_2 - n \cdot f_z^{0.78}.$$

Let  $\lambda_1 \neq 0$  and  $\lambda_2 = 0$ . Then we get:

$$\begin{cases} -\alpha_1 \cdot f_z^{-1} \cdot n^{-1} + 2.13 \cdot \alpha_2 \cdot f_z^{-0.06} \cdot n^{2.13} - \lambda_1 \cdot \beta_1 = 0; \\ -\alpha_1 \cdot f_z^{-1} \cdot n^{-1} - 0.06 \cdot \alpha_2 \cdot f_z^{-0.06} \cdot n^{2.13} - 0.3 \cdot \lambda_1 \cdot \beta_1 = 0. \end{cases}$$

After conversion and substitution we get:

$$0.7 \cdot \alpha_1 f_z^{-1} n^{-1} + 0.609 \cdot \alpha_2 f_z^{-0.06} n^{2.13} = 0 \rightarrow f_z^{0.94} n^{3.13} = \frac{-0.7 \cdot \alpha_1}{0.609 \cdot \alpha_2}.$$

The last expression has no physical meaning because  $n > 0$  and  $f > 0$ .  
Let  $\lambda_1 = 0$  and  $\lambda_2 \neq 0$ . Then we get:

$$\begin{cases} 2.19 \cdot f_z^{-0.06} \cdot n + 0.78 \cdot \lambda_2 \cdot \beta_2 = 0; \\ f_z^{0.78} = \beta_2. \end{cases}$$

Converting this system gives the equation:

$$f_z^{-0.06} \cdot n^{2.13} = \frac{-0.78 \cdot \lambda_2 \cdot \beta_2}{2.19}.$$

Since,  $\lambda_2 \geq 0$ , the last expression has no physical meaning.

Let  $\lambda_1 = 0$  and  $\lambda_2 = 0$ . Then we have:

$$\begin{cases} n \cdot f_z^{0.3} = \beta_1 \rightarrow n = \beta_1 \cdot f_z^{-0.3} = 43.47 \cdot 0.498^{-0.3} = 53.58 \text{ min}^{-1}; \\ f_z^{0.78} = \beta_2 \rightarrow f_z = \beta_2^{1.28} = 0.498 \frac{\text{mm}}{\text{tooth}}. \end{cases}$$

This solution satisfies the initial constraints and affords a minimum of the objective function in the range of admissible values.

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## CONCLUSIONS

1. An analytical apparatus and algorithmic basis for solving problems in the field of metal cutting on metal-cutting machines has been developed. The main conceptual principle underlying this monograph is the representation of the problem of part shaping as a process of simultaneous realization of sub processes:

- a) elastic-plastic deformation (EPD) during removal of excess layer of machined material;
- b) wear and blunting of the cutting tool;
- c) formation of the part machined surface with set parameters of dimensions, quality and accuracy.

2. A logical scheme for determining the angular parameters of the working part of cutting tools has been created in the following sequence:

*Surfaces and edges of the cutting wedge → basic shaping movement's → coordinate planes → angular parameters of the tool in statics → angular parameters of the tool in kinematics*

Typical problems of determining the static angles of the cutter during workpiece cutting off technological operation, kinematic angles during metric thread cutting and kinematic angles during machining of curvilinear surface of the part are solved.

3. Analysis of physical phenomena during cutting in the applied aspect has been made. The basic interrelation of the stress and strain state in the transition zone of machining is carried out based on I.A. Time classical works, the founder of the cutting theory and K. A. Zworykin's law. The basic hypothesis is the representation of the EPD transition zone as a single shear plane (in conditions of rather high speeds of cutting and getting of the continuous chip).

4. On the basis of indicators of chip shrinkage coefficient and relative shear as EPD criteria, estimation of energy parameters of cutting

process – total and specific work of cutting forces is given. A typical task of determining the specific work of cutting forces in the EPD transition zone is solved taking into account the analytically and quantitatively determined values of the force and deformation rate of the processed material. The tasks formulations for determining various energy parameters of the cutting process, including the determination of the generalized empirical dependence of the cutting force tangential component by graph-analytical method are presented.

5. The research of different mechanisms of tool wear and their influence on tool life dependences (in coordinates "tool life period-cutting speed") is carried out. It is noted that the form of the tool life dependence and its extreme character depends on the interaction of wear mechanisms (for example, the interaction of abrasive and adhesive wear), the laws of the build-up on the front surface, etc. A typical task of determining the optimum period of cutter durability with the use of the criterion of optimum wear (N.N. Zorev's method) is solved. At definition of the total period of tool life for re-sharpening the assumption of constancy of height of a chamfer of wear along the main blade of the tool is accepted.

6. A complex toolkit for the calculation of the cutting conditions on the metal-cutting machine tools of turning, milling and drilling types has been introduced. For rough turning of a stepped shaft, a system of technical constraints and the target function are formed: productivity of the machining process through the index of machine time spent on cutting the unit length of the part. Optimal data by linear programming method by graph-analytical method were obtained. Constructed polygon of admissible decisions may serve as a basis for choosing the most rational equipment with given initial data.

7. Software-methodical complex of solving optimization problems by geometrical programming method (MGP) is developed. The effectiveness of this method, especially for solving nonlinear type problems is shown. All components of optimization problem can be expressed quantitatively in the form of generalized positive polynomials – posinomials. The

constructive point here is finding, first of all, an extreme of target function and relative contribution of each component to its value, and then optimal values of optimized variables. The availability of information on the relative contribution of the various components to the optimality of the design decision makes it possible to identify the direction for improving the technological systems of machining.

8. For the case of high-dimensional problems, when the system of linear equations does not have a single solution, this monograph implements computational procedures, reducing such problems to MGP of zero degree of difficulty. It presents techniques for combining some terms of the target function based on their weights, as well as partial invariance techniques that take into account the dominant terms of the target function. Importantly, these techniques allow one to realize stepwise improvement of the initial solution by replacing the dominant term.

9. At the same time, when the proposed MGP methods of problem reduction to zero degree of difficulty are inapplicable in case of absence of initial solution necessary for estimation of dual variables, the author suggests the procedure of problem solution of the first degree of difficulty. In order to realize this procedure, a dual problem, which is considered as an optimization problem is composed and by searching for an extreme of values of a single redundant, dual variable is solved.

10. Efficient methods of numerical solution of design problems with the help of GP method of the first-degree difficulty which reduces general labor intensity of optimization calculations are presented. The use of partial invariance method in the optimization problems of cutting modes by the example of a typical problem of single-tool machining by cutting is considered. Mechanism of realization of this method is reduced to bringing the optimization problems to the zero degree of difficulty and consists in iteration using property of geometrical inequality in minimizing point for searching optimal (close enough to optimal) values of controlled variables for design problem. To investigate two-pass turning, the Wilde method is applied. It makes it possible to solve a system of linear equations with

respect to the least costly tool component. In this dual formulation, the system of equations does not have a single solution.

11. The monograph presents a software-methodological complex for solving optimization problems by the Lagrange multipliers method (LMM). This method of analytical research of extreme technological problem gives possibility to define qualitative picture of optimal solution behavior when the structure and parameters of the problem are changing. It is also effectively used in development of man-machine procedures of extreme search in which analytical means of results analysis for numerical solution of initial optimization problem are used.

12. For finish turning, when tool life must be a multiple of cutting time when machining one part and active force constraint on the tangential component, a technique for entering an additional equation reflecting the number of parts machined during the tool life period is proposed. When solving these equations together, you can determine two pairs and between which on the constraint line lies the point of optimum of cutting modes. Such a problem arises when it is necessary to change the tool and the need to strictly maintain the constraint on the cutting force. In this case, the designer is faced either with the requirement to reduce the tool life while leaving the cutting time unchanged or to increase the cutting time while leaving the tool life unchanged.

13. The paper presents an example of solving a multi-criteria optimization problem by the climb-down method. It has been tested experimentally and allows using multi-criteria optimization for different types of machining. The formulation of the problem of search for an optimum under many efficiency criteria, which are contradictory and reach a maximum at various points of the set of admissible alternatives, is given. The climb-down method in the optimization of finish turning according to the criteria of minimum machine time and minimum camber deflection of the part is used. When solving the optimization problem, linear and geometrical programming methods as well as the numerical dichotomy method were used. The linear programming method was used when

considering the problem of finding the minimum shaft camber deflection under the constraints of tool cutting capability and allowable surface roughness value. The MGP of zero degree of difficulty in such statement is inapplicable, because of the appearance of negative dual weights, which contradicts the initial restriction of the MGP. At the same time, the problem of finding the minimum machining time is solved by MGP of the first degree of difficulty. The peculiarity of the solution algorithm is integration with numerical optimization method (dichotomy, golden section or Fibonacci numbers), which gives rather close to optimal solution with standard unimodal optimization programs.

14. As calculations have shown, the desire to minimize machining time and shaft camber deflection leads to two sets of optimized parameters. In order to find a compromise variant of machining modes realization, a procedure of selecting climb-down (for example, by comparing the machine time obtained as a result of unicriteria optimization with the table time value according to the standards) is proposed.

## Appendix 1

### *Conditional shear plane. Zworykin theorem*

The formation process of continuous chips in the free cutting scheme (planing type) is characterized by the following processing scheme (Fig. A.1).

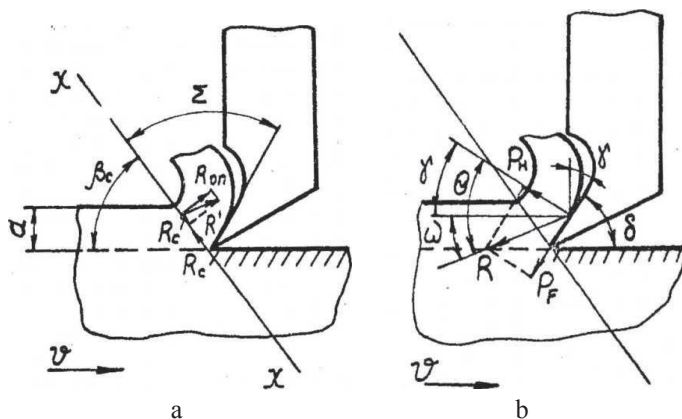


Fig. A1. Free cutting scheme (planing type): a – system of forces acting in the shear plane; b – system of forces acting on the rake surface of the cutter

The initial empirical basis for K.A. Zvorykin was the facts obtained in the course of experiments by the Russian scientist I.A. Time and others:

1. Plastic deformation in front of the tool-cutting wedge is limited by a certain surface (in simplification – by a certain plane X-X);
2. Plastic deformations in the cut layer under the action of the tool do not spread uniformly over the entire section and within a certain angle  $\Sigma$  (Fig. A.1, a);
3. The inclination of the X-X plane to the cutting plane is limited for free cutting by narrow range of the angle  $\beta_s = 15^\circ \dots 45^\circ$  (Fig. A1, a);

4) The plane X-X, called the conditional shear plane, moves parallel to itself in the process of moving the tool.

The initial theoretical basis for determining the shear angle was the set of primary assumptions (axioms) put forward by K.A. Zworykin:

1) Shear (shipping) cut layer occurs in a single plane – the shear plane;

2) The shear in the plane in which the shear resistance is minimal is carried out;

3) The values of shear stress in the shear plane  $\tau_s$ , friction angle  $\theta$  and cutting angle  $\delta$  (or rake angle  $\gamma$ ) do not depend on the position of the shear plane, i.e. from the angle  $\beta_s$ ;

4. The deformation state of the cut metal layer is practically two-dimensional, which is correct in the case when the cut width  $b_c$  is much greater than the cut thickness  $a_c$ ;

5. Internal normal stresses under the action of forces in shear planes  $R_c$  are insignificant and can be neglected.

The procedure for theoretical determination of the shear angle  $\beta_s$  includes the same sequence of logical and analytical transformations.

For a chip formation process with a single shear plane, the expression for the force  $R$  (chip formation) depends on the size and properties of the chip material:  $R = f(\tau_s, b_c, a_c, \mu, \delta, \beta_s)$ . Here  $\mu = \tan \theta$  is the coefficient of friction.

The cutting force is balanced by the resistance of the material being processed to the cutting movement with the force  $R'$  (Fig. A.1, a). If we neglect the internal normal stresses in the shear plane X-X from the action of the compression force  $R_c$ , we obtain the following expression for determining chip formation force  $R$ :

$$\tau = \frac{R_c}{F_{sp}} = \frac{R \cdot \cos(\omega + \beta_s) \cdot \sin \beta_s}{a_c \cdot b_c}; \quad R = \frac{\tau_s \cdot a_c \cdot b_c}{\cos(\omega + \beta_s) \cdot \sin \beta_s}, \quad (47)$$

where  $F_{sp}$  – shear plane area ( $F_{sp} = L_{ps} \cdot b_c$ ). Using assumptions about plane deformation in the transition zone and independence of  $\tau_s$  from  $\beta_s$  we represent the expression (47) in the following form:

$$R = \frac{C_s}{\cos(\omega + \beta_s) \cdot \sin \beta_s},$$

where  $C_s$  is some constant characterizing the processing conditions.

The assumption of shear in the plane of minimum shear resistance can be mathematically interpreted:

$$\begin{aligned} \frac{dR}{d\beta_s} = 0; \quad \frac{dR}{d\beta_s} &= \frac{C_s}{\cos(\omega + \beta_s) \cdot \sin \beta_s} = C_s \left( \frac{\cos \beta_s}{\cos(\omega + \beta_s)} \right)' = \\ &= C_s \cdot \left\{ \frac{1}{(\cos(\omega + \beta_s))^2} \cdot \left[ \cos(\omega + \beta_s) \cdot \left( -\frac{\sin \beta_s}{\sin^2 \beta_s} \right) - \left( -\frac{\sin(\omega + \beta_s)}{\sin \beta_s} \right) \right] \right\} = \\ &= C_s \cdot \left[ \frac{1}{(\cos(\omega + \beta_s))^2} \cdot \left( \frac{\cos(\omega + 2 \cdot \beta_s)}{-\sin^2 \beta_s} \right) \right] = 0. \end{aligned}$$

As a result of solving this equation, the dependence of the shear angle  $\beta_s$  on the angle of action  $\omega$  is obtained (Zvorykin's formula):

$$\omega + 2 \cdot \beta_s = 90^\circ; \quad \beta_s = \frac{90^\circ - \omega}{2}.$$

## REFERENCES

1. Bobrov V.F. Fundamentals of the theory of metal cutting. – M.: Fundamentals of the theory of metal cutting. – M.: Mechanical Engineering, 1975. – 273 p.
2. Makarov A.D. Optimization of cutting processes. – M.: Mechanical Engineering, 1976. – 278 p.
3. Reznikov A.N. Thermophysics of cutting. – M.: Mechanical Engineering, 1969. – 288 p.
4. Armarego I.J. Metal Cutting / I.J. Armarego, R.H. Brown – M.: Mechanical Engineering, 1977. – 325 p.
5. Granovsky G.I. Metal Cutting: A Textbook for Engineering and instrumentation universities / G.I. Granovsky, V.G. Granovsky. – M.: Higher. Shk., 1985. – 304 p.
6. Krol O.S., Zarubitskiy E.U., Kisilev V.N. Theory of Metal Cutting in Examples and Tasks. – K.: UMK VO, 1992. – 124 p.
7. Yascheritsyn P.I. Fundamentals of cutting materials and cutting tools / P.I. Yascheritsyn, M.L. Eremenko, M.I. Zhigalko. – Minsk: Ab. School, 1975 – 528 p.
8. Yashcheritsyn P.I. Fundamentals of cutting materials and cutting tools / P.I. Yascheritsyn, M.L. Eremenko, M.I. Zhigalko. – Minsk: Ab. School, 1981. – 560 p.
9. Basics of metal theory: Pidruchnik for vishch. navch. mortgages / M.P. Mazur, Yu.M. Vnukov, V.L. Dobroskok, V.O. Pledge, Yu.K. Novoselov, F.Ya. Yakubov; pid zag. ed. M.P. Masuria. – Lviv: Noviy svit, 2009. – 422 p.
10. Solonenko V.G. Metal Cutting and Cutting Tools: Textbook / V.G. Solonenko, A.A. Ryzhkin. – M.: INFRA-M, 2013. – 416 p.
11. Krol O. S., Hmelovskij G. L. Optimizacija i upravlenie processom rezanija: uchebnoe posobie. Kiev: UMK VO, 1991. – 140 p.
12. Krol O.S. Parametric modeling of metal-cutting machines and tools. Monograph. – Lugansk: EUNU, 2012. – 116 p.

13. Krol O.S. Methods and procedures of 3D-modeling of metal-cutting machine tools and instruments. Monograph: Severodonetsk, EUNU, 2015. – 120 p.
14. Krol O.S., Sokolov V.I. Trivimirnoe modelirovanie metallorizalnyh verstativnyh and iinstrumentalnogo equipments. Sievierodonetsk: SNU named after V. Dahl, 2016. – 160 p.
15. Feldshtein E.E., Kornievich M.A. Cutting tool. Tutorial. – Minsk: New knowledge, 2007. – 400 p.
16. Abramov F.N. Handbook of metal cutting / F.N. Abramov, V.V. Kovalenko, V.E. Lyubimov, etc. – K.: Tekhnika, 1983. – 255 p.
17. Poduraev V.N. Technology of physico-chemical processing methods. – M.: Mechanical Engineering, 1985. – 264 p.
18. Krol, O.S., Burlakov, E.I. Modelirovanie shpindel'nogo uzla obrabativayushchego zentra [Modeling of spindle node for machining center]. Visnik NTU "HPI". – [Bulletin of the NTU "HPI"]. – 2013, 11(985). – P. 33–38.
19. Krol, O.S., Krol, A.A., Burlakov, E.I. (). Tverdotel'noe modelirovanie i issledovanie shpindel'nogo uzla obrabatyvayushchego tsentra. [Modeling of spindle node for machining center]. Visnik NTU "HPI". – [Bulletin of the NTU "HPI"]. – 2013, 16(989). – P. 14–18.
20. Krol, O., Sokolov, V.: Optimization of Processing Modes on Multioperational Machines Using Two-parameter D-Partitions. 2020 International Russian Automation Conference (RusAutoCon), pp. 57-62. IEEE (2020). <https://doi.org/10.1109/RusAutoCon49822.2020.9208120>
21. Krol, O., Porkuian, O., Sokolov, V., Tsankov, P.: Vibration stability of spindle nodes in the zone of tool equipment optimal parameters. Comptes rendus de l'Académie bulgare des Sciences 72(11), 1546-1556 (2019). <https://doi.org/10.7546/CRABS.2019.11.12>
22. Krol O.S. Methods and procedures for optimizing cutting conditions. Monograph. – Lugansk: Publishing house VDEUNU, 2013 – 260 p.
23. The processing of metals by cutting: Handbook of technology / A.A. Panov, Anikin V.V., Boym N.G. et al. – Moscow: Mechanical Engineering, 1988. – 736 p.
24. Time I.A. Resistance of metals and wood to cutting. St. Petersburg, 1870.

25. Jacobs G.Yu. Cutting optimization / G.Yu. Jacobs, E. Jacob, D. Cohan. – M.: Mechanical Engineering, 1981. – 279 p.
26. Krol O, Sokolov V. Modeling of carrier system dynamics for metal-cutting machines/IEEE Proceedings 2018 International Russian Automation Conference (RusAutoCon) P. 1 – 5.  
<https://doi.org/10.1109/rusautocon.2018.8501799>
27. Krol, O.S., Suhorutchenko, I.A. (2014). Trehmernoje modelirovanie obrabatyivayuschiy tsentra SVM1F4 v KOMPAS 3D [Solid modeling of machining centre SVM1F4 in KOMPAS 3D]. Eastern-European Journal of Enterprise Technologies], 4/7(70), 13–18.
28. Krol O.S., Krol A.A., Sindeeva E.V. Modeling of the design of the four-shaft in the CAD APM "WinMachine" / Resource-saving technologies of production and processing of materials in mechanical engineering/ Collection of scientific papers. – Lugansk: Publishing House of Volodymyr Dahl East Ukrainian National University, 2008. – P. 139–143.
29. Krol O. Modeling of vertical spindle head for machining center / O. Krol, V. Sokolov, P. Tsankov // Journal of Physics: Conference Series 1553 (2020) 012012. – VSPID-2019. <http://doi.org/10.1088/1742-6596/1553/1/012012>
30. Zvorykin K.A. Work and effort required to separate metal chips. – M.: Russian litho-typography, 1893. – 76 p.
31. Krol O., Sokolov V. Parametric modeling of machine tools for designers. – Sofia: Prof. Marin Drinov Academic Publishing House of Bulgarian Academy of Sciences, 2018. – 112 p.  
<https://doi.org/10.7546/PMMTD.2018>
32. Krol O., Sokolov V. Modelling and calculation of machine gear cutting tools for designers. – Sofia: Prof. Marin Drinov Academic Publishing House of Bulgarian Academy of Sciences, 2021. – 150 p.  
<https://doi.org/10.7546/MCMGCTD.2021>
33. Shevchenko S., Mukhovaty A., Krol O. (2020) Gear Transmission with Conic Axoid on Parallel Axes. In: Radionov A., Kravchenko O., Guzeev V., Rozhdestvenskiy Y. (eds) Proceedings of the 5th International Conference on Industrial Engineering (ICIE 2019). ICIE 2019. Lecture Notes in Mechanical Engineering. Springer, Cham. pp. 1-10.  
[https://doi.org/10.1007/978-3-030-22041-9\\_1](https://doi.org/10.1007/978-3-030-22041-9_1)

34. Sokolov V, Porkuian O, Krol O and Baturin Y 2020 Design Calculation of Electrohydraulic Servo Drive for Technological Equipment. Advances in Design, Simulation and Manufacturing III. DSMIE 2020 LNME 1, 75 [https://doi.org/10.1007/978-3-030-50794-7\\_8](https://doi.org/10.1007/978-3-030-50794-7_8)

35. Sokolov V., Krol O., Stepanova O. Mathematical model of the automatic electrohydraulic drive with volume regulation // TEKA. Commission of Motorization and Energetics in Agriculture. – Vol. 17. – N 1. – 2017. – Lublin – Rzeszow, Poland. – P. 27-32.

36. Krol O.S., Krol A.A. Calculation of compliance SF68VF4 machine dynamics shaping and modeling // Vestnik SevNTU. Ser. Engineering and Transportation. – 2011, is. 117. – P. 81-84.

37. Zorev N.N. Research of elements of mechanics of the cutting process. – Moscow: Mashgiz, 1952. – 364 p.

38. Zhed V.P. Method for calculating optimal cutting conditions / V.P. Zhed, A.I. Soson, V.M. Bashkov // Bulletin of mechanical engineering. – 1979. – Is. 9. – P. 43–45.

39. Krol, O. S., Sokolov, V. I. Metody i procedury inzhenerenogo prognozirovaniya v stankostroenii [Methods and procedures of engineering forecasting in machine tool building]. – Lugansk: VNU, 2017. – 114 p.

40. Krol, O. S., Sokolov, V. I. Metody i procedury racional'nogo vybora v stankostroenii [Methods and procedures of rational choice in machine tool construction]. Lugansk: Publishing House of Volodymyr Dahl East Ukrainian National University, 2017. – 112 p.

41. Khmelovsky G.L., Krol O.S., Surnin Y.M. Fundamentals of Technological Design Automation. Textbook. – Kyiv: UMK VO, 1989. – 188 c.

42. Krol O., Shevchenko S., Sukhorutchenko I and Lysenko A. (2014). 3D-modeling of the rotary table for tool SVM1F4 with non-clearance worm gearing. TEKA Commission of Motorization and Energetic in Agriculture, 14, 1, 126-133.

43. Krol, O., Sokolov, V.: Research of modified gear drive for multioperational machine with increased load capacity. Diagnostyka 21(3), 87-93 (2020). <https://doi.org/10.29354/diag/126026>

44. General machine-building standards of time and cutting conditions for the regulation of work performed on universal and multi-purpose machines

with numerical control. Part II Standards for cutting conditions. – M.: Economics, 1990. – 474 p.

45. Reference technologist-machine builder. Ed. A. G. Kosilova. – M.: Mechanical Engineering, 1986. Vol. 2. – 446 p.

46. Reference machine-builder rationing: In 2 vols. – T.2 / Ed. E.M. Struzhestrah. – Moscow: State Publishing House, 1961. – 892 p.

47. Krol O. S. (2012) Construction of parametrical models of belt transmissions with the use of the system APM WINMACHINE. East.-Eur. J. Enterp. Technol., 2/7(62), 37–51.

48. Krol O.S., Shevchenko S.V., Sindeeva E.V., Pokintelytsa N.I. Design of mechanical gears of metal-cutting machines with the help of a system APM WinMachine. Handbook. – Lugansk: Publishing House of Volodymyr Dahl East Ukrainian National University, 2007. – 200 p.

49. Krol O.S. Tools of rational choice of technological systems of machining / Bulletin of NTUU (KPI), is. 32. Mechanical Engineering series, 1997. – P. 157 – 161.

50. Krol O.S. Elements of the methodology of rational choice of technological systems of mechanical processing / Bulletin of NTUU (KPI), is. 32. Mechanical Engineering series, 1997. – P. 198–204.

51. Krol O., Sokolov V. Rational choice of machining tools using prediction procedures/EUREKA: Physics and engineering, is. 4, 2018. – P. 14–20. <https://doi.org/10.21303/2461-4262.2018.00667>

52. Goransky G.K. Calculation of cutting modes using electronic computers. – Mn .: State Publishing House of the BSSR, 1963. – 192 p.

53. Ishutkin V.I. Calculation of cutting conditions on automatic and semi-automatic machines. – Bulletin of mechanical engineering, 1969. Is. 7. – P. 21–27.

54. Krol O., Tsankov P., Sokolov V. Rational choice of two-support spindles for machining centers with lubrication system/EUREKA: Physics and Engineering, is. 3, 2018. – P. 52–58. <https://doi.org/10.21303/2461-4262.2018.00648>

55. Zhuk K.D., Krol O.S., Timchenko A.A. Prognostic analysis of objects of new technology and technology in the tasks of system design. – Kiev, 1984. – 27 p. (Preprint / Academy of Sciences of the Ukrainian SSR, Institute of Cybernetics; 84–54).

56. Krol O.S., Krol A.A. Parametrization of the transverse layouts of the drive of the main motion / Tool reliability and optimization of technological systems. Digest of scientific works – Kramatorsk: is. 24, 2009. – P. 164–168.

57. Krstić, M. (2014). Rational choice theory and addiction behaviour. Market-Tržište, 26 (2), 163-177.

58. O Krol and V Sokolov. 3D modelling of angular spindle's head for machining centre / J. Physics: Conf. Series **1278** (2019) 012002. – VSPID-2018. <https://doi.org/10.1088/1742-6596/1278/1/012002>

59. Krol O., Belkov M. Study dynamics machining centre SF68VF4 / Teka Komisji Motoryzacji i Energetyki Rolnictwa, OL PAN, 2014, Vol.14, is.2, Lublin, Poland. – P. 59–67.

60. Krol O.S., Osipov E.I., Krol A.A. Simulation of IT-1 machine tool drive in APM WinMachine environment / Bulletin of Volodymyr Dahl East Ukrainian National University. – Lugansk: SNU, is. 2 (191). Part 1, 2013. – P. 112–115.

61. Krol O, Sokolov V. Research of toothed belt transmission with arched teeth. Diagnostyka. 2020;21(1):15-22. <https://doi.org/10.29354/diag/127193>

62. Krol O, Sokolov V. Selection of worm gearing optimal structure for machine rotary table. Diagnostyka. 2021;22(1):3-10. <https://doi.org/10.29354/diag/129949>

63. Krol O.S., Sokolov V.I. 3D Modeling Of Machine Tools For Designers. – Sofia: Prof. Marin Drinov Academy Publishing House of Bulgarian Academy of Sciences, 2018. – 140 p. [https://doi.org/10.7546/3D\\_momtfd.2018](https://doi.org/10.7546/3D_momtfd.2018)

64. Jacobs G.Yu. Cutting optimization / G.Yu. Jacobs, E. Jacob, D. Cohan. – M.: Mechanical Engineering, 1981. – 279 p.

65. Krol O., Sokolov V. Rational choice of machine tools for designers. – Sofia: Prof. Marin Drinov Academic Publishing House of Bulgarian Academy of Sciences, 2019. – 113 p. <https://doi.org/10.7546/RCMTD.2019>

66. Rekleitis G. Optimization in technology / G. Rekleitis, A. Reyvindran, K. Ragsdel. – M.: Mir, 1986. Vol. 2. – 320 p.

67. Krol O.S. Express procedure for studying the cost of two-pass processing / University news. Engineering, 1990, is. 1. – P. 122–124.

68. Krol O., Sokolov V. Selection of machine tools optimal cutting modes for designers. – Sofia: Prof. Marin Drinov Academic Publishing House of Bulgarian Academy of Sciences, 2020. – 240 p. <https://doi.org/10.7546/SMTOCMD.2020>
69. Gilman A.M. Optimization of processing modes on metal cutting machines / A.M. Gilman, L.A. Brahman, D.I. Batishchev. – M.: Mechanical Engineering, 1972. – 188 p.
70. Krol O. Engineering forecasting of machine tools for designers. – Sofia: Prof. Marin Drinov Academic Publishing House of Bulgarian Academy of Sciences, 2019. – 114 p. <https://doi.org/10.7546/EFMTD.2019>
71. Khmelovsky G.L. Krol O.S. Analytical optimization of cutting conditions in CAD of technological processes / IV All-Union Coordination Meeting on the automation of design work in mechanical engineering. - Minsk: ITK AN BSSR, 1989. – P.153–156.
72. Wilde D. Optimal design. – M.: Mir, 1981. – 272 p.
73. Krol O. Machine tool spindle dynamics for designers. – Sofia: Prof. Marin Drinov Academic Publishing House of Bulgarian Academy of Sciences, 2020. – 143 p. <https://doi.org/10.7546/MTCDD.2020>
74. Krol, O., Sokolov, V., Tsankov, P., Logunov, O.: Modelling of machining center vibration stability by the D-partitions method. Journal of Physics: Conference Series 1745 012085 (2021). <https://doi.org/10.1088/1742-6596/1745/1/012085>
75. Goransky G.K. Calculation of cutting modes using electronic computers. – Mn.: State Publishing House of the BSSR, 1963. – 192 p.
76. Krol O., Sokolov V. (2019) Parametric Modeling of Gear Cutting Tools. In: Gapiński B., Szostak M., Ivanov V. (eds) Advances in Manufacturing II. Lecture Notes in Mechanical Engineering. Springer, Cham. [https://doi.org/10.1007/978-3-030-16943-5\\_1](https://doi.org/10.1007/978-3-030-16943-5_1)
77. Krol O., Sokolov V. (2019) Parametric Modeling of Transverse Layout for Machine Tool Gearboxes. In: Gapiński B., Szostak M., Ivanov V. (eds) Advances in Manufacturing II. MANUFACTURING 2019. Lecture Notes in Mechanical Engineering. Springer, Cham. [https://doi.org/10.1007/978-3-030-16943-5\\_11](https://doi.org/10.1007/978-3-030-16943-5_11)
78. Duffin R. Geometric programming / R. Duffin, E. Peterson, K. Zener. – M.: Mir, 1972. – 12 p.

79. Ivchenko T.G. Using the geometric programming method to calculate the optimal cutting conditions for turning // Scientific Herald of the DSEA. – Is. 2 (8E), 2011. – P. 110–116.

80. Krol O.S., Khmelovsky G.L., Peycheva V.V. Optimization of the cutting process by geometric programming / Technology and automation of machine building. Is. 44, 1989. – P. 44 – 47.

81. Krol O.S., Khmelovsky G.L., Peycheva V.V. Methods of combining and dual geometric programming of optimal cutting conditions / Technology and automation of mechanical engineering. Vol. 46, 1990. – P. 58 – 63.

82. Ivchenko T.G. Dvokhkriterialnaya optimizatsiya mode v rizannya pid hour of processing of chavuniv with tools from overhard materials / T.G. Ivchenko, C.V. Polyakova // Progressive technology and engineering systems. – Donetsk: DonNTU, 2011. – Is. 41. – P. 152–158.

83. Krol O.S. Optimization of two-pass processing method geometric programming / University news. Engineering, 1990. Is. 1. – P. 122–124.

84. Krol O.S., Shevchenko S.V., Sokolov V.I. Design of metal-cutting tools in the middle of APM WinMachine. Textbook. – Lugansk: SNU, 2011. – 388 p.

85. Krol O.S., Khmelovsky G.L. Optimization of the cutting process by the method of “ideal point” / Technology and automation of mechanical engineering. Vol. 49, 1992. – P. 34–38.

86. Velikanov K.M. Calculations of the economic efficiency of new technology: Handbook. – M.: Mechanical Engineering, 1975. – 432 p.

87. Ivchenko T.G. Optimization of cutting conditions for fine and fine turning by geometric programming / T.G. Ivchenko, E.E. Shalskaya // Progressive technologies and engineering systems. – Donetsk: DonNTU, 2010. – Is. 39. – P. 91-97.

88. Krol O, Sokolov V, Golubenko A. Modification of rack-and-pinion transmission design with increased resource. Diagnostyka. 2022; 23(1):2022105. <https://doi.org/10.29354/diag/145967>

89. Ermer D. Optimization of the turning mode in several passes in the presence of restrictions / D. Ermer, S. Kramodikadio // Design and technology of mechanical engineering. – 1981. – No. 4. – P. 281–289.

90. Ivata K. Optimization of cutting conditions for multi-pass operations taking into account the probabilistic nature of machining processes / K. Iwata, Yu. Murotsu, F. Oba // Design and Engineering Technology. – 1977. – Is. 1. – P. 152–159.

91. Shevchenko S., Muhovaty A., Krol O. Geometric Aspects of Modifications of Tapered Roller/ Procedia Engineering 150 (2016) 1107 – 1112. <https://doi.org/10.1016/j.proeng.2016.07.221>

92. Shevchenko S., Muhovaty A., Krol O. Gear Clutch with Modified Tooth Profiles / Procedia Engineering 206 (2017) 979–984. <http://doi.org/10.1016/j.proeng.2017.10.581>

93. Krol O.S., Khmelovsky G.L. The method of successive climb-down in the problems of optimization of cutting processes / Technology and automation of mechanical engineering. Vol. 48, 1991. – P. 28–31.

94. Krol O, Shumakova T, Sokolov V. Design metal cutting instruments by dint of system of KOMPAS. – Lugansk: V. Dahl EUNU, 2013. – 144 p.

95. Krol O.S. Methods and procedures for the dynamics of spindle nodes. Monograph. – Lugansk: EUNU, 2014. – 154 p.

96. Krol O.S., Shevchenko S.V., Sokolov V.I. Design of metal-cutting tools in the middle of APM WinMachine. Textbook. – Lugansk: SNU, 2011. – 388 p.

97. Rakhmatulin R.R. Automation of designing cutting conditions on multi-purpose machines / R.R. Rakhmatulin, A.I. Serdyuk, A.O. Cossacks. V.N. Kuzmin. – Software products and systems. Is. 1, 2013. – P. 21–28.

98. Nerubaschenko, A.A., Krol, O.S., Krol, A.A. Creation of a database of parametric models for machine parts in the APM Base module // Bulletin of SevNTU, 2010, is. 107. – P.107-109.

99. Krol O.S., Guzieva Yu.A. Construction of parametric models of relieves tooth/Progressive directions of development of machine-instrument-making industries and transport. Materials of the international scientific and technical conference of students, graduate students and young scientists. May 19–22, 2009, Sevastopol. – P. 19–22.

100. Krol O.S., Krol A.A. Building spindle models of a multi-purpose lathe in the APM WinMachine environment/Bulletin of Volodymyr Dahl East Ukrainian National University. – Lugansk: SNU, is. 3 (145). Part 2, 2010. – P.143–148.

101. Krol O.S., Krol A.A. Modeling the construction of a rack and pinion gear using APM Studio/ Lugansk: Publishing House of Volodymyr Dahl East Ukrainian National University, 2011, is. 2 (156), part 2. – P. 173–178.

102. Solomentsev Yu.M. Optimization of mechanical processes of machining and assembly in mass production / Yu.M. Solomentsev, A.M. Basin. – M.: NIIMASH, 1977. – 72 p.

103. Krol O.S. Rational choice of supports spindles of machine tools // Tool reliability and optimization of technological systems. Collection of scientific papers. – Kramatorsk: V. 21, 2007. – P. 164 – 169.

104. Krol O.S. The use of predictive procedures in the tasks of selecting the supports of spindle units of machine tools. // Tool reliability and optimization of technological systems. Collection of scientific papers. – Kramatorsk: V. 23, 2008. – P. 153–158.

105. Krol O.S. Methods and procedures for the dynamics of spindle nodes. Monograph. – Lugansk: Publishing house VDEUNU, 2014. – 154 p.

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